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## ITTC Quality System Manual

### Recommended Procedures and Guidelines

#### Procedure

### Extrapolation for Direct Stability Assessment in Waves


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## Extrapolation for Direct Stability Assessment in Waves

### 1. PURPOSE OF PROCEDURE

This procedure provides detailed guidance on extrapolation of ship motion numerical simulation data to estimate probabilities of large roll angle and capsizing. Envelope Peak Over Threshold (EPOT) method is described for estimation of rate of exceedance of large roll angles. Split-time / Motion Perturbation method (MPM) is described for estimate rate of capsizing events in waves. The procedure also highlights the limitations of these methods.

Current procedure describes extrapolation for a given sea state, speed and heading.

### 2. INTRODUCTION

#### 2.1 Formulation of the Problem

Complexity of physical phenomena related to stability in waves makes numerical simulation the only tool, suitable for detail assessment.

Extrapolation allows extending existing numerical simulation data beyond observation. For example, no exceedances of 40° was not observed over 10 hr of simulation, but there were some 30° and 35° exceedances. Extrapolation methods allow estimating an exceedance rate of 40° by using the existing data rather than run more simulations. Indeed, the extrapolation carries more statistical uncertainty than an estimate based on direct direct observation. Extrapolation is the only technique, capable of estimating probability of capsizing, as the latter is too rare to observe in realistic conditions.

The IMO second generation intact stability criteria allow use of extrapolation as one of the tools for direct stability assessment (paragraph 3.5.5.4.1 of Annex of SDC 7/WP.6).

#### 2.2 Theoretical Background


Extrapolation is based on two theorems on extreme values. The first extreme value theorem (Fisher-Tippett-Gnedenko) proves that a distribution of the largest value in a sample has a limit in the form of a Generalized Extreme Value (GEV) distribution. The second extreme value theorem (Pickands-Balkema-de Haan) shows that the GEV distribution can be approximated by a Generalized Pareto Distribution (GPD) above a large-enough threshold, (see, e.g. Coles 2001). Extrapolation application can be developed using both GEV and GPD distributions.

Peak-over-threshold (POT) method is a generic extrapolation method based on GPD (Pickands, 1975). A key feature of the POT extrapolation is that it can capture the nonlinearity of the large amplitude response, such as that caused by the changes in the restoring at large roll angles and in waves.

The tail ( $y > u$ ) of *any* distribution can be approximated with a GPD above a sufficiently large threshold. The GPD is defined by three numbers – a shape parameter  $\xi$ , a scale parameter  $\sigma$ , and threshold value  $u$  – and has the following form for  $y > u$ :

$$\text{pdf}(y) = \begin{cases} \frac{1}{\sigma} \left(1 + \xi \frac{y-u}{\sigma}\right)^{-\left(1+\frac{1}{\xi}\right)} & \xi \neq 0 \\ \frac{1}{\sigma} \exp\left(-\frac{y-u}{\sigma}\right) & \xi = 0 \end{cases} \quad (1)$$

$$\text{cdf}(y) = \begin{cases} 1 - \left(1 + \xi \frac{y-u}{\sigma}\right)^{-1/\xi} & \xi \neq 0 \\ 1 - \exp\left(-\frac{y-u}{\sigma}\right) & \xi = 0 \end{cases} \quad (2)$$

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The objective of the present application is to estimate a rate of exceedance  $\hat{\lambda}(c)$  of a target value  $c > u$  above the threshold  $u$ :

$$\hat{\lambda}(c) = \hat{\lambda}(u) \widehat{\text{cdf}}(c) \quad (3)$$

where  $\hat{\lambda}(u)$  is the rate of upcrossing of the threshold  $u$ , estimated directly from the time series.

For application of a GPD, three parameters must be found: shape  $\xi$  and scale  $\sigma$  and threshold  $u$ . The scale parameter  $\sigma$  is positive, while the shape parameter  $\xi$  can be either positive or negative. A negative shape parameter imposes a limitation on the expressions in parenthesis of equations and formally introduces a right bound to the distribution:

$$\text{pdf}(y) = 0, \quad \text{if } y > u - \frac{\sigma}{\xi} \quad \text{and } \xi < 0 \quad (4)$$

The shape parameter defines the type of tail: heavy, exponential, or light, as shown in Figure 1.

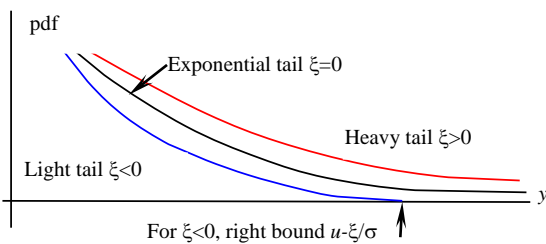


Figure 1 Types of tails per GPD approximation

The roll restoring arm ( $\overline{GZ}$ ) curves of most ships have a limited range of stability, leading to the appearance of an unstable equilibria at the angle of vanishing stability, as well a maximum value of  $\overline{GZ}$ . This configuration leads to a heavy tail after the maximum of the  $\overline{GZ}$  curve, which switches to a light tail in the immediate vicinity

of the angle of vanishing stability. Figure 2 illustrates the configuration, see (Belenky, et al 2019) for details.

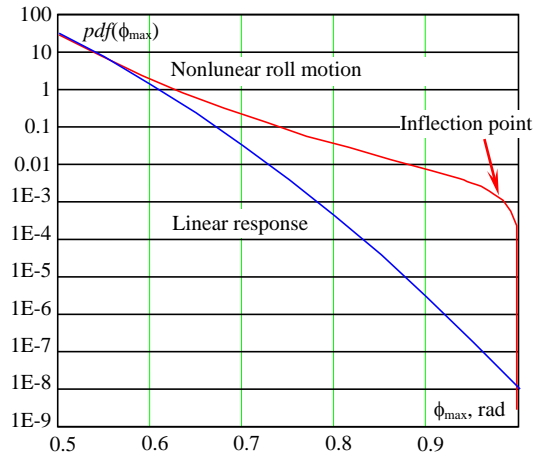



Figure 2 Configuration of a trail of roll peak distribution

The standard POT method is only applicable to independent data points, while the roll motions of a ship are correlated because of the ship's inertia, correlated wave excitation, and “memory” in the hydrodynamic forces. The application of POT, therefore, requires an extraction of independent points from the time history, a process known as “de-clustering.”

Fitting an envelope to the time history of the roll motion, as illustrated in Figure 3, is a convenient way to de-cluster the data, as the peaks of the envelope of the roll response are sufficiently far from each other to provide the necessary independence. The use of an envelope to de-cluster the roll motion provides the additional letter in the acronym of the method, so POT becomes EPOT – Envelope Peaks Over Threshold

Decustering with the envelope uses a property of relatively narrow band spectrum of roll motion that is usually correct for beam, following and stern-quartering seas. As the roll spectrum grows wider in head and oblique seas, the

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envelope peaks may need to be checked for decorrelation as recommended in Section 3 of this document.

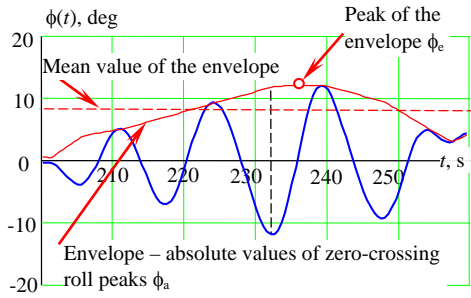


Figure 3 De-clustering using an envelope

Estimation of the rate of capsizing requires an additional step: calculation of capsizing metric with MPM. The roll rate is perturbed until capsizing is observed (see Figure 4). The difference between the roll rate at upcrossing and the roll rate when capsizing is observed is the metric, as this difference indicates “the distance to trouble.”

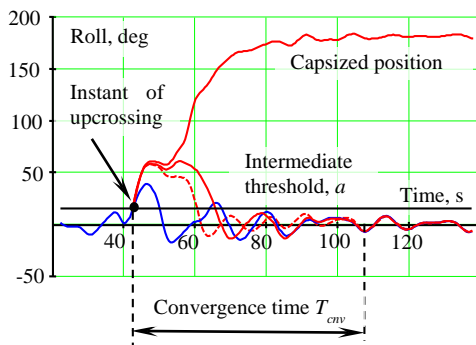


Figure 4 Motion perturbation for computing the capsizing metric

The capsizing metric is a random number showing how likely capsizing is at a given instant of time. It has been demonstrated that the tail of the MPM capsizing metric is exponential. Algorithm of calculation of the capsizing metric and its extrapolation are given in Section 4 of this document.

Finally, relation between the probability of and time needs to be mentioned. If there is a requirement for estimation of a probability of exceedance of a certain roll angle or a capsizing, a time of exposure,  $T_e$  must be specified. Then the probability that at least one exceedance (or capsizing) will occur during the time  $T_e$  can be estimated as:

$$\hat{P}(T_e) = 1 - \exp(-\hat{\lambda}(c)T_e) \quad (5)$$

where  $\hat{\lambda}(c)$  is an estimate of exceedance or capsizing rate.


### 3. ROLL EXCEEDANCE RATE

#### 3.1 Data Requirement

The minimum required combined length of time histories of roll motion is 40 hours. While the bulk of statistical validation was performed for 100 hours time histories (Weems et al. 2019), a successful test for 40 hours is described in (Belenky 2020), subsection 5.3.3.

It may be possible to run the EPOT extrapolation for shorter time. It will require favorable comparison of the extrapolation based of shorter simulation with the extrapolation using at least 40 hours of the simulation time history.

The simulation time histories should include sufficient information on nonlinearity of the roll motion. At least 5% of the envelope peaks should exceed a half of the angle of maximum of the GZ curve in calm water. Caution has to be exercised when applying the method to oblique heading, while this requirement may be slightly relaxed in stern-quartering seas (Belenky et al. 2018).

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### 3.2 Data Preparation

It is convenient to present roll motion time history data as a set of independent records. Length of each record should be about 30 minutes.

Zero-crossing peaks are extracted from each record. Absolute values of these peaks  $\phi_a$  form the envelope as shown in Figure 3.

Mean value for the envelope is estimated for each record. Mean-crossing peaks of envelope  $\phi_e$  are collected for further processing.

In the case of oblique seas, when the spectrum of roll motion is widening, an additional check for the data independence needs to be performed. Time is recorded together with each peak of roll motion and retained for the mean-crossing envelope peaks  $\phi_e$ .

Decorrelation time  $T_d$  (time duration to reach independence) is estimated as described in the subsection 3.2.3 of the ITTC Procedure 7.5-02-01-08 (see Figure 5). Estimation of auto-covariance function  $\hat{R}$  is described in the subsection 3.2.1 of the above procedure.

The time between the envelope peaks should be more than the decorrelation time  $T_d$ . If one or several peaks of the envelope are located closer than  $T_d$ , the largest among them is used for the further processing.

### 3.3 Fitting the Distribution

The tail of the distribution of a large roll angle is supposed to be heavy (see Figure 2). When the shape parameter  $\xi > 0$  and threshold value  $u = \sigma/\xi$ , the GPD is equivalent to a Pareto distribution with scale  $y_m = \sigma/\xi$  and shape  $\alpha = 1/\xi$ :

$$\text{pdf}(y) = \frac{\alpha y_m^\alpha}{y^{\alpha+1}} \quad (6)$$

The conditional probability of exceedance of a target value  $y$  associated with dynamic stability failure is expressed as:

$$P(Y > y | Y > u) = \left(\frac{u}{y}\right)^\alpha = \left(\frac{y}{u}\right)^{-\frac{1}{\xi}} \quad (7)$$

Here, the threshold  $u$  does not have to be the same as in the GPD case. A method for finding the threshold and estimating the shape parameter is proposed in Belenky et al. (2018), which is based on Beirlant, et al. (2004), Dupuis and Victoria-Feser (2006), and Mager (2015).

To extrapolate with equation (7), the threshold is found from applicability considerations so only one parameter needs to be fitted.

The input data for fitting consists of  $N$  independent envelope peaks  $\phi_e$  (Figure 3). After the independence of the peaks has been established, there is no need to keep track of the number of records and the time when a peak has been recorded. The method is applied to a sample sorted in descending order – a.k.a. order statistics:


$$Y = \text{sort}_{\text{desc}}(\phi_e) \quad (8)$$

The Hill estimator provides the shape parameter  $\xi$  – for the case of Pareto  $\xi > 0$ :

$$\hat{\xi}_k = \frac{1}{k} \sum_{i=1}^k \log\left(\frac{Y_i}{Y_k}\right) \quad (9)$$

where the index  $k$  refers to the number of upper order statistics used in the estimation. Mager (2015) suggests the first index  $k = \min(40, 0.02N)$ , while the last (largest) value for the index taken as  $0.2N$ .

The threshold  $u$  is found by an index that corresponds to a minimum of the mean squared prediction error function:

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$$\hat{\Gamma}(k) = \frac{1}{\hat{\xi}_k^2 k} \sum_{i=1}^k \frac{\left( \log\left(\frac{Y_{i-1}}{Y_{k-1}}\right) + \hat{\xi}_k \log\left(\frac{i}{k+1}\right) \right)^2}{\left( \sum_{j=i}^k j^{-2} \right)} + \frac{2}{k^2} \sum_{i=1}^k \frac{\left( \log\left(\frac{i}{k+1}\right) \right)^2}{\left( \sum_{j=i}^k j^{-2} \right)} - 1 \quad (10)$$

Once the index  $k$  corresponding to a minimum of  $\hat{\Gamma}$ , is found, the threshold is set as:

$$u = Y_k \quad (11)$$

The extrapolated estimate of the exceedance rate of target value  $c$  can be computed as:

$$\hat{\lambda}(c) = \hat{\lambda}(u) \left(\frac{c}{u}\right)^{-1/\hat{\xi}} \quad (12)$$

where  $\hat{\lambda}(u)$  is the rate of upcrossing of threshold  $u$ , estimated as:

$$\hat{\lambda}(u) = \frac{k}{T_{tot}} \quad (13)$$

where  $T_{tot}$  is the total time of all the records available.

### 3.4 Assessment of Uncertainty

The uncertainty of extrapolation coming from finite volume of the data sample can be evaluated with confidence interval, containing the true value with confidence probability  $\beta$ .

The confidence interval for the extrapolated value is computed assuming a normal distribution for the estimate of the shape parameter  $\hat{\xi}$ . Its variance estimate is expressed as:

$$\widehat{Var}(\hat{\xi}) = \frac{\hat{\xi}^2}{k} \quad (14)$$

The boundaries of the confidence interval of the estimate are:

$$\hat{\xi}_{up,low} = \hat{\xi} \pm K_\beta \sqrt{\widehat{Var}(\hat{\xi})} \quad (15)$$

where  $K_\beta$  is a half of a non-dimensional confidence interval computed as a normal quantile of  $0.5(1+\beta)$ . For  $\beta=0.95$ ,  $K_\beta=1.96$ .

The number of upcrossings of the threshold  $k$  has a binomial distribution with the estimate of a parameter

$$\hat{p} = \frac{k \Delta t}{T_{tot}} \quad (16)$$

where  $\Delta t$  is time increment used in the simulation. The confidence interval for  $\hat{\lambda}(u)$  is computed using normal approximation for binomial distribution. The variance of the number of upcrossings  $k$  is estimated as:

$$\hat{V}_k = \frac{\Delta t}{T_{tot}} \hat{p}(1 - \hat{p}) \quad (17)$$

Then the confidence interval for  $\hat{\lambda}(u)$  is computed as:


$$\hat{\lambda}_{up,low}(u) = \frac{N_{aU} \pm K_\beta \sqrt{\hat{V}_k}}{T_{tot}} \quad (18)$$

Boundaries for the extrapolated value are computed through the lower and upper boundaries of the upcrossing rate estimate  $\hat{\lambda}_{low,up}(u)$  and the shape parameter estimate  $\hat{\xi}_{low,up}$ :

$$\begin{cases} \hat{\lambda}_{low}(c) = \hat{\lambda}_{low}(u) \left(\frac{c}{u}\right)^{-1/\hat{\xi}_{low}} \\ \hat{\lambda}_{up}(c) = \hat{\lambda}_{up}(u) \left(\frac{c}{u}\right)^{-1/\hat{\xi}_{up}} \end{cases} \quad (19)$$

Equations (19) contain a product of the boundaries of two estimates. If the desired confidence probability for the entire extrapolated estimate  $\hat{\lambda}(c)$  is to be  $\beta=0.95$  then the confidence probabilities for each estimate  $\hat{\lambda}(u)$  and  $\hat{\xi}$  must be set as:

$$\beta_1 = \sqrt{\beta} = \sqrt{0.95} = 0.975 \quad (20)$$

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In order to account for the difference in the confidence probability,  $K_\beta$  is set to 2.236 in equations (15) and (18).

## 4. CAPSIZING RATE

### 4.1 General

Primary application area for the Split-Time / Motion Perturbation Method (MPM) is estimation of probability of capsizing. In general, the split-time method is intended for estimating the probability of complex and rare physical phenomena in which the physics of the problem changes with the extreme response, such as that caused by capsizing in dead ship conditions or pure loss of stability in stern quartering / following seas.

### 4.2 Data Requirements

The minimum required combined length of time histories of roll motion is 40 hours. While the bulk of statistical validation was performed for 100 hours time histories (Belenky et al. 2018), a successful test for 40 hours is described in (Belenky 2020), subsection 5.3.4.

It may be possible to run the split-time /MPM method using shorter simulation time. It will require favorable comparison of the extrapolation based of shorter simulation with the extrapolation using at least 40 hours of the simulation time history.

There is no requirement on how large roll motions should be. The treatment of nonlinearity is included in computation of the capsizing metric (or the metric of capsizing likelihood).

### 4.3 Data Preparation


The metric of capsizing likelihood is computed at the instant of upcrossing of an intermediate roll threshold  $a$ .

The choice of the intermediate threshold  $a$  is done based on consideration of computational efficiency. If the threshold is set too high, there will be too few upcrossings available for the metric computations and additional simulation time may be required. If the threshold is set too low, there will be many upcrossings leading to dependent metric values. Thus, many of these values will be rejected during the de-clustering procedure, which will decrease computational efficiency. 7-10 upcrossings per an independent record of 30 minutes long was found to be acceptable.

The evaluation of the metric consists from the following steps.

1. Compute the time instant of up crossing  $t_{Ui}$  and the roll rate at the instant of upcrossing  $\dot{\phi}_{Ui}$  using linear interpolation between the roll angle and rate data points, respectively. Similarly compute values of heave, pitch and their derivatives at the instant of upcrossing.
2. Compute perturbed solution for  $\phi_0 = a$  and  $\dot{\phi}_0 = \dot{\phi}_{Ui} + \Delta\dot{\phi}$ , while using the values for pitch and heave, computed at the previous step, as the initial conditions for pitch and heave equations respectively. If capsizing is not observed, the perturbed solution converges with the unperturbed solution (see Figure 4), the convergence time  $T_{cnv}$  is defined when the difference between perturbed and unperturbed solution does not exceed a given value for a given number of points.
3. The next perturbation is carried out for  $\dot{\phi}_0 = \dot{\phi}_{Ui} + 2\Delta\dot{\phi}$ , keeping the rest of the initial conditions the same. The procedure is repeated for  $3\Delta\dot{\phi}$ ,  $4\Delta\dot{\phi}$ , ..., until the capsizing is



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observed as shown in Figure 4 for  $\dot{\phi}_0 = \dot{\phi}_{Ui} + m\Delta\dot{\phi}$ , where  $m$  is the number of iterations.

4. Once capsizing is observed for  $\dot{\phi}_0 = \dot{\phi}_{Ui} + m\Delta\dot{\phi}$ , the critical roll rate is computed as  $\dot{\phi}_{Ci} = \dot{\phi}_{Ui} + (m - 1)\Delta\dot{\phi}$ , convergence time  $T_{cnv}$  for the penultimate iteration is recorded for further use in the declustering procedure.
5. Metric for the  $i$ -th upcrossing is computed as:

$$y_i = 1 + \dot{\phi}_{Ui} - \dot{\phi}_{Ci}; \quad i = 1, \dots, N_U \quad (21)$$

where  $\dot{\phi}_{Ci}$  is the critical roll rate calculated for the  $i$ -th upcrossing, and  $\dot{\phi}_{Ui}$  is the roll rate observed at the  $i$ -th upcrossing,  $N_U$  number of observed upcrossings.

The up crossings of a level usually comes in groups, the metric values are clustered, meaning that the metric values within a same cluster may be dependent. A de-clustering procedure is used to ensure independence of the collected metric values. The cluster is defined as a group of the metric values  $y_i$ , corresponding to up crossings that are closer than respective convergence time durations  $\{y_i\}_{i=b_j}^{i=e_j}$ , where  $b_j$  and  $e_j$  are indexes of the beginning and end of  $j$ -th cluster. The de-clustered values are determined as maximum value within each cluster:

$$x_j = \max\left(\{y_i\}_{i=b_j}^{i=e_j}\right), j = 1 \dots N \quad (22)$$

#### 4.4 Fitting the Distribution

As demonstrated in Belenky et al (2018), the independent large values of the metric (21) can be approximated with an exponential tail. Fitting a tail involves finding a threshold, after which the exponential approximation is applicable. Two options are described below: one is the prediction error criterion (Mager 2015) and the

other is the goodness-of-fit test (Stephens, 1974).

To use the prediction error technique, the data needs to be sorted in descending order. A mean squares prediction error function is defined as (Mager 2015):

$$\hat{\Gamma}(k) = \frac{1}{\hat{\gamma}_k^2} \sum_{i=1}^k \left(\frac{k+1}{i} - 1\right)^{-1} \left(x_{i,k} + \hat{\gamma}_k \log\left(\frac{i}{k+1}\right)\right)^2 + \frac{2}{k} \sum_{i=1}^k \left(\frac{k+1}{i} - 1\right)^{-1} \left(\log\left(\frac{i}{k+1}\right)\right)^2 - 1, \quad (23)$$

where  $k$  is an index corresponding to a candidate threshold, while  $\hat{\gamma}_k$  is exponential distribution parameter estimated on the subset of the data points indexes from 1 to  $k$ .

$$k \in [\min(40, 0.02N); 0.2N] \quad (24)$$

$$\hat{\gamma}_k = \frac{1}{k} \sum_{j=1}^k (x_j - x_k) \quad (25)$$

The resulting threshold  $u$  (not to be confused with intermediate threshold  $a$ , used earlier) is found where the mean squares prediction error function experiences a global minimum.


To use the goodness-of-fit technique, the data needs to be sorted in ascending order. The candidate threshold  $u$  is searched for the data point indexes  $k$  within the following range:

$$k \in [0.7N; N - 10] \quad (26)$$

For a candidate threshold  $u$ , there are  $n$  points available for fitting ranging from 10 to  $0.3N$ . The data points  $v_j$  above the candidate threshold are  $u$  defined as:

$$v_j = x_j - u; \quad j = 1, \dots, n \quad (27)$$

The exponential distribution parameter,  $\hat{\gamma}_u$ , estimated for the candidate threshold  $u$

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$$\hat{\gamma}_u = \frac{1}{n} \sum_{j=1}^n v_j \quad (28)$$

The test statistic for the null hypothesis is defined as:

$$D^* = \left( D - \frac{0.2}{n} \right) \left( \sqrt{n} + 0.26 + \frac{0.5}{\sqrt{n}} \right) \quad (29)$$

where  $D = \max\{D^+, D^-\}$  with:

$$D^+ = \max_{j=1, \dots, n} \left| \frac{j}{n} - z_j \right| \quad (30)$$

$$D^- = \max_{j=1, \dots, n} \left| z_j - \frac{j-1}{n} \right| \quad (31)$$

$$z_j = 1 - e^{-\frac{v_j}{\hat{\gamma}_u}} \quad (32)$$

The critical values for the test statistic  $D^*$  were tabulated by Stephens (1974) and placed in Table 1. For a range of candidate thresholds, the test statistic  $D^*$  could then be computed and a  $p$ -value of the test for the exponential distribution be calculated. Since Stephens (1974) tabulated critical values for only a few significance levels, the  $p$ -value can be taken as the significance level whose critical value is the largest but still smaller than  $D^*$ .

Table 1 Critical Values for Goodness-of Fit Method

Level of significance, $p$	0.01	0.02 5	0.05	0.10
$D^*$	1.30 8	1.19 0	1.094	0.990
Level of significance, $p$	0.15	0.20	0.25	0.30
$D^*$	0.92 6	0.88 0	0.835	0.795
Level of significance, $p$	0.35	0.40	0.45	0.50
$D^*$	0.76 6	0.73 6	0.710	0.685

A final threshold can be selected as the largest  $u$  above which the thresholds have their associated  $p$ -value larger than certain significance

level. A significance level of 5% seems reasonable but it has been found that for the extrapolation to work, it should be larger, say 10% or above.

The extrapolated estimate of the capsizing rate is computed as

$$\hat{\lambda} = \hat{\lambda}(u) \exp\left(-\frac{1-u}{\hat{\gamma}}\right) \quad (33)$$

where the estimate of the parameter  $\hat{\gamma}$  corresponds to the chosen threshold  $u$ ,  $\hat{\lambda}(u)$  is the rate of upcrossing of threshold  $u$ , estimated with equation (13).

#### 4.5 Assessment of Uncertainty


The uncertainty of the estimation of the capsizing rate is driven by the finite volume of the data available for extrapolation. The confidence interval for the extrapolated value is computed assuming a normal distribution for the estimate of the parameter  $\hat{\gamma}$ . Its variance estimate is expressed as:

$$\widehat{Var}(\hat{\gamma}) = \frac{\widehat{Var}(x-u)}{n} \quad (34)$$

where  $\widehat{Var}(x-u)$  is the variance, estimated for the points above the threshold  $u$  while  $n$  is the volume of sample above the threshold  $u$ . The boundaries of the confidence interval of the estimate are:

$$\hat{\gamma}_{up,low} = \hat{\gamma} \pm K_\beta \sqrt{\widehat{Var}(\hat{\gamma})} \quad (35)$$

where  $K_\beta$  is a half of a non-dimensional confidence interval computed as a normal quantile of  $0.5(1+\beta_2)$ , where  $\beta_2$  is the confidence probability for the parameter, computed as  $\beta_2 = \sqrt{\beta}$ , while is the confidence interval accepted for the entire extrapolated estimate.

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Boundaries for the extrapolated estimate are computed through the lower and upper boundaries of the upcrossing rate estimate  $\hat{\lambda}_{low,up}(u)$ , (see equation 18) and the parameter estimate  $\hat{\gamma}_{up,low}$ :

$$\begin{cases} \hat{\lambda}_{low}(c) = \hat{\lambda}_{low}(u) \exp\left(-\frac{1-u}{\hat{\gamma}_{low}}\right) \\ \hat{\lambda}_{up}(c) = \hat{\lambda}_{up}(u) \exp\left(-\frac{1-u}{\hat{\gamma}_{up}}\right) \end{cases} \quad (36)$$

Similar to equations (19), the equation (36) contain a product of the boundaries of two estimates. If the desired confidence probability for the entire extrapolated estimate  $\hat{\lambda}(c)$  is to be  $\beta = 0.95$  then the confidence probabilities for each estimate  $\hat{\lambda}(u)$  and  $\hat{\gamma}$  must be set as shown in equation (20)


## 5. LIST OF SYMBOLS

$c$	Target for extrapolation
cdf	cumulative density function
$N$	Total number of independent data points
$\hat{P}(T_e)$	Estimate of probability of at least one exceedance or capsizing occurs during the time of exposure
pdf	probability density function
$T_d$	Time of decorrelation, s
$T_e$	Time of exposure, s
$T_{tot}$	Total time of available simulation, s
$u$	Threshold value
$Var()$	Variance operator
$\beta$	Confidence probability
$\gamma$	Parameter of exponential distribution
$\lambda(c)$	Exceedance or capsizing rate $s^{-1}$
$\lambda(u)$	Exceedance of a threshold $u$ , $s^{-1}$
$\xi$	Shape parameter of Generalized Pareto distribution or Pareto distribution
$\sigma$	Scale parameter of Generalized Pareto Distribution
$\phi$	Roll angle, deg

$\phi_a$	Roll amplitude (absolute values of roll peaks), deg
$\dot{\phi}$	Roll rate, rad/s
$\hat{\phantom{x}}$	Estimate (“hat” above a symbol)
$_{up}$	Upper boundary of confidence interval
$_{low}$	Lower boundary of confidence interval

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## Appendix A. CALCULATION EXAMPLE

### A.1. Input Data

Both extrapolation procedures are demonstrating using ONR (US Office of Naval Research) tumblehome topside configuration (Bishop, et al 2005). Principle dimensions and other general input data are placed to Table A1, while the lines are shown in Fig. A1

Fast volume-based simulation tool (Weems, et al 2018) was used to generate sample data. Simulation included 3 degrees of freedom: heave-roll-pitch, with body-nonlinear formulation for hydrostatic and Froude-Krylov forces, while appropriate coefficients were used for the rest of forces. Wave environment was represented by long-crested irregular waves generated with Bretschneider spectrum recommended by ITTC 1978.

Table A1 Principle Input Data

Length BP, m	154
Breadth molded, m	18
Draft amidships, m	5.5
KG, m	7.5
Displacement, MT	8675.6
GM, m	2.2
Speed, kt	6
Heading, deg	45
Significant wave height, m	9
Total volume of sample, hr	40
Duration pf each record, hr	0.5

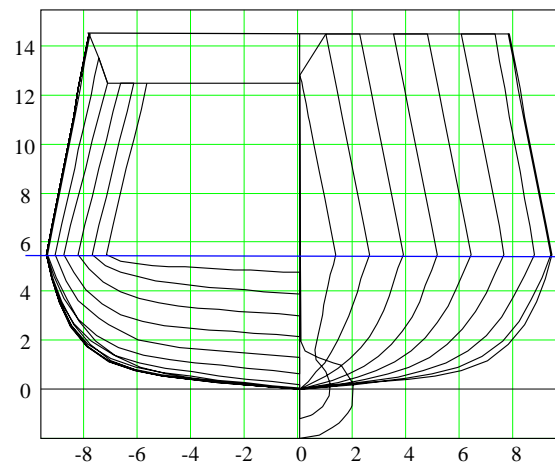



Figure A1 ONR Tumblehome Topside Configuration

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## A.2. Results of EPOT Extrapolation

Example calculation were carried out for irregular waves with modal period of 15 s. Sample record is shown in Figure A2. Maximum observed angle over 40 hours of simulation was 28.1°.

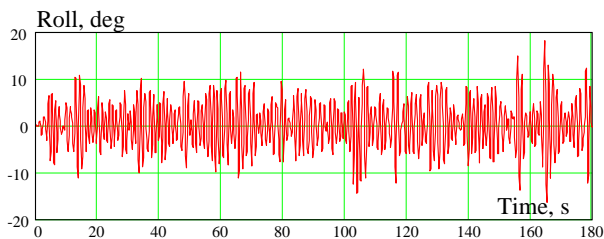


Figure A2 Sample Record

There were total of 22318 zero-crossing peaks in 40 records, 2294 peaks were found to be independent, using envelope-base de-clustering procedure. The first index to search threshold ( $k = \min(40, 0.02N)$ ) were taken as 40, while the last index ( $0.2N$ ) equals 459, leave 420 potential thresholds. The mean squared prediction error function shown in Figure A6.

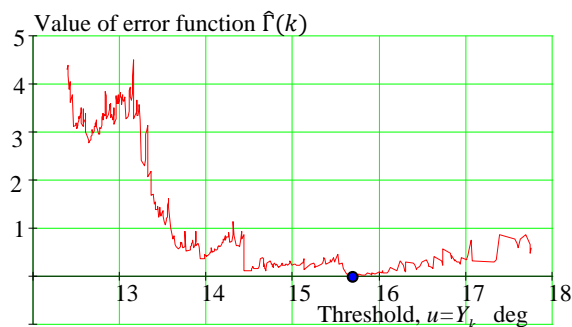


Figure A3 Mean squares prediction error function

The index, corresponding to the minimum error value was found to be 347, corresponding to the angle of  $u=15.68$  deg. Number of point available for fitting the tail is  $k=112$ .

The shape parameters of Pareto distribution was estimated as  $\hat{\xi} = 0.121$  using equation (9). Rate of upcrossing through the level  $u$  is estimated as  $\hat{\lambda}(u) = 7.91 \cdot 10^{-4} s^{-1}$  with equation (13). The final results - rate of exceeding of 50 degrees is  $\hat{\lambda}(c = 50) = 5.46 \cdot 10^{-8} s^{-1}$ .

Assessment of uncertainty requires calculation of variances of the shape parameters of Pareto distribution using formula (14):  $\widehat{Var}(\hat{\xi}) = 1.309 \cdot 10^{-4}$  and variance of the the number of upcrossing using formula (17)  $\widehat{V}_k = 111.956$ . The final result is shown in Figure A4.

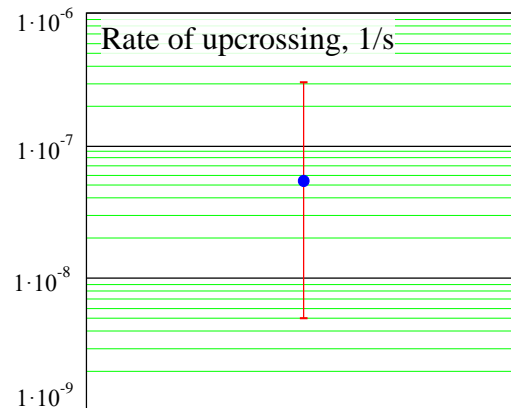


Figure A4 Estimate for Rate of Upcrossing

## A.3. Results of Split-Time/ MPM Extrapolation

Example calculation were carried out for irregular waves with the modal period of 14s. The maximum observed roll angle over 40 hours of simulation was 39.4 degrees. The intermediate threshold was chosen at 12 degrees, resulting in 527 upcrossing over 80 records. The de-clustering procedure produced 321 independent values of the metric, ranging from 0.344 rad/s to 0.767 rad/s.

For the choice of the threshold with the Prediction Error Criterion, the mean squares prediction error function is shown in Figure A5. Other intermediate numerical results are given in Table A2

The final extrapolation results for both methods of fitting is shown in Figure A6.

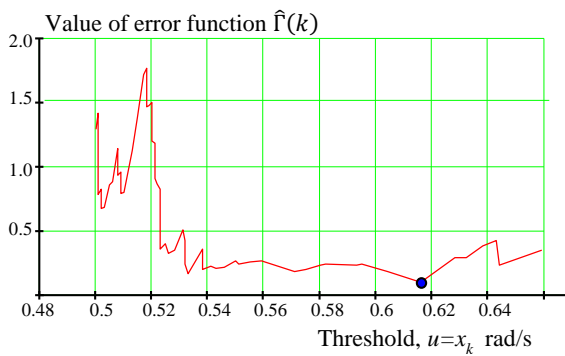


Figure A5 Estimate for Rate of Upcrossing

Table A2 Intermediate Results of Fitting Exponential Tail

Method	Prediction Error Function	Goodness-of-Fit Test	
		p=0.1	p=0.2
Threshold, $u$	0.616	0.515	0.518
Available points	13	49	45
Parameter $\hat{\gamma}$ rad/s	0.058	0.062	0.064
Variance of $\hat{\gamma}$ (rad/s) <sup>2</sup>	$1.78 \cdot 10^{-4}$	$9.41 \cdot 10^{-5}$	$9.41 \cdot 10^{-5}$
$\hat{P}(y > 1)$	$1.38 \cdot 10^{-3}$	$3.86 \cdot 10^{-4}$	$5.13 \cdot 10^{-4}$
Capsizing rate $\hat{\lambda}$ , s <sup>-1</sup>	$1.27 \cdot 10^{-7}$	$1.34 \cdot 10^{-7}$	$1.63 \cdot 10^{-7}$

Table A2 Intermediate Results of Fitting Exponential Tail (Cont.)

Method	Goodness-of-Fit Test		
	p=0.3	p=0.4	p=0.5
Threshold, $u$	0.518	0.604	0.604
Available points	45	13	13
Parameter $\hat{\gamma}$ rad/s	0.064	0.07	0.07
Variance of $\hat{\gamma}$ (rad/s) <sup>2</sup>	$1.04 \cdot 10^{-4}$	$1.93 \cdot 10^{-4}$	$1.93 \cdot 10^{-4}$
$\hat{P}(y > 1)$	$5.13 \cdot 10^{-4}$	$3.48 \cdot 10^{-4}$	$3.48 \cdot 10^{-4}$
Capsizing rate $\hat{\lambda}$ , s <sup>-1</sup>	$1.63 \cdot 10^{-7}$	$3.19 \cdot 10^{-7}$	$3.19 \cdot 10^{-7}$

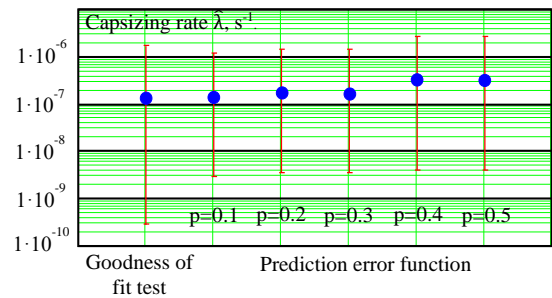


Figure A6 Estimate for Rate of Capsizing