ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Predicting the Occurrence and Magnitude of Parametric Rolling

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-07	Loads and Responses
7.5-02-07-04	Stability
7.5-02-07-04.3	Predicting the Occurrence and Magnitude of Parametric Rolling

Updated / Edited by	Approved
Stability in Waves Committee of 29th ITTC	29th ITTC 2021
Date 05/2020	Date 06/2021

Table of Contents

Predicting the Occurrence and Magnitude of Parametric Rolling

1. PURPOSE OF PROCEDURE

This procedure provides detailed guidance on the numerical methods for predicting the occurrence and magnitude of parametric rolling. It also highlights the limitations of these methods.

2. INTRODUCTION

2.1 Historical Note

The problem of parametric rolling of ships has been recognised for more than half a century (e.g. Paulling & Rosenberg, 1959), first experimental observation (Paulling et al. 1972). The fundamental dynamics that create this behaviour is considered in our days as reasonably clarified. Large as well as smaller ships have been investigated on the basis of theory and experiment, by and large for a following seas situation. In this state, it is easier to satisfy one of the necessary conditions namely that the frequency of encounter with waves whose length is similar or larger than the ship length is comparable to twice the roll natural frequency of ship. However, an accident with a post-Panamax containership in predominantly head seas, which led to extreme roll angles and accelerations with substantial loss and damage to containers stowed on the deck, was also attributed to parametric rolling (France et al. 2003).

Model tests and full scale observations have shown that parametric rolling can occur not only in long-crested (longitudinal) head and following seas, but also at slightly oblique heading angles with and without directional wave energy spreading.

2.2 Physical Background

The physical phenomenon is based on successive alterations of the restoring lever between crests and troughs, exhibited by many ships in steep longitudinal waves. These set up a mechanism of internal (parametric) excitation in roll. There is a clear analogy with a simple oscillator governed by the so-called Mathieu equation with damping (see e.g. Shin et al 2004). Brief description of physical background of parametric rolling is available from paragraph 2.4.1 of Annex of SDC 7/INF.2.

2.3 Other Instruments

ABS Guide (ABS 2019) is applicable to container carriers. It contains criteria to determine susceptibility and severity of parametric rolling in head and following seas as well as recommendations for numerical simulations with hybrid potential-flow code and development of operational guidance. Theoretical background of the Guide is described in (Shin, et al 2004).

Parametric roll is included as a failure mode in current development of the second generation IMO intact stability criteria. To improve efficiency, the criteria are presented in the tiered form: vulnerability levels 1 and 2 (see section 1.1.3 of Annex of SDC 7/WP.6) and direct stability assessment (see Paragraph 3.1.1 of Annex SDC 7/WP.6). The level 1 is the simplest and the most conservative aiming to exclude obviously irrelevant cases. The level 2 vulnerability is more complex, but less conservative. Direct stability assessment is recommended if the level 2 criteria have indicated vulnerability.

2.4 Structure of this Procedure

Accurate prediction of occurrence and magnitude of parametric roll for a given ship may be labour extensive. Sometimes, however, the state-of-the-art accuracy is not required, when a quick assessment has been requested.

Following examples of 2nd generation IMO intact stability criteria, this procedure is structured in three levels, reflecting required resources:

Level 1: Mostly closed form formulae, no professional software required.

Level 2: Professional software for calculation of righting arm curve in waves is required; additionally, some in-house software development.

Level 3: Use of potential flow / hybrid codes is expected. This level represents the state-of-the-art of assessment of magnitude and occurrence of parametric roll.

3. PREDICTION OF OCCURRENCE AND MAGNITUDE OF PARAMETRIC ROLLING

3.1 Mathematical Models for Level 1 Prediction

The simplest mathematical model of parametric roll is based on a single degree-of-freedom equation of roll in following or head seas. The model must include variation of stability. Nonlinearity of roll restoring is essential for assessment of the magnitude of parametric roll. Damping may be either linear or nonlinear; wave direction is assumed to be exactly head or following, so there is no direct wave excitation in roll:

$$(I_x + A_{44})\ddot{\phi} + B_{\phi}(\dot{\phi}) + W\overline{GZ}(\phi, t) = 0$$

where,

ϕ	roll angle
I_x	mass moment of inertia in roll
A_{44}	added mass in roll
B_{ϕ}	roll damping (function of roll rate)
W	weight
$\overline{GZ}$	righting arm (function of roll angle and time)

Parametric rolling typically occurs in various combinations of ship speed and wave encounter frequency ω_e, provided that the resulting frequency of encounter is near to $(2/n)$ times the natural frequency of roll $\omega_\phi=2\pi f_\phi$, where n is any integer. The symbol ω is used for circular frequencies in rad /s, while f is a frequency in Hz. The practical relevance of the n=1 scenario ("principal resonance", $\omega_e \approx 2\omega_\phi$) is well established for ships. The n=2 scenario ("fundamental resonance") is also believed to be of interest although with a lower probability of occurrence in a seaway, as the frequency range, where parametric resonance can occur, is narrower.

The build-up of parametric rolling requires a threshold wave height in addition to fulfilment of the above condition of frequencies. The minimum wave height is determined in principle by two factors: the degree of fluctuation of roll restoring due to wave passage, and the ship's roll damping which is speed dependent. This threshold is higher for fundamental resonance. The damping is a key design parameter for the development of parametric rolling. None of the current state-of-the-art computational programs can claim to calculate the roll damping accurately for any given vessel including all roll damping devices. The restoring moment variation may be estimated based on balancing the vessel in the undisturbed wave at different roll angles and positions in the wave. It is noted, however, that effects related to forward speed and disturbed waves may influence the roll restoring moment. Longitudinal waves having a length of the order of the ship length will typically lead to the largest fluctuations of the roll restoring moment.

Magnitude of parametric rolling depends on nonlinearity of the restoring moment. Change of an instantaneous $\overline{GM}$ leads to the change of instantaneous natural frequency, eventually breaking the parametric resonance condition. That limits parametric excitation and establishes magnitude of parametric rolling.

3.2 Level 1 Prediction of Occurrence of Parametric Roll

For the prediction of parametric rolling due to principal resonance the following simple rule may be applied, which is based on consideration of the asymptotic stability of the upright state of a ship in regular longitudinal waves (Francescutto *et al.* (2004); Spyrou, (2005)).

$$\ddot{\phi} + 2\delta\dot{\phi} + \frac{W\overline{GM}(t)}{I_x + A_{44}}\phi = 0$$

where, δ is a dimensional coefficient of linear roll damping

$$\delta = \frac{W\overline{GM}(t)}{2(I_x + A_{44})}$$

If $\overline{GM}$ varies on the wave between $\overline{GM}_{\min}$ and $\overline{GM}_{\max}$ for different positions of the considered wave along the hull; and the scaled amplitude of variation of metacentric height, defined as follows:

$$h = \frac{\overline{GM}_{\max} - \overline{GM}_{\min}}{2\overline{GM}_{\mathrm{m}}}$$

where $\overline{GM}_{\mathrm{m}} = (\overline{GM}_{\max} + \overline{GM}_{\min})/2$ is the mean metacentric height of the ship for the considered regular wave (with a wavelength of about length of the ship). If h exceeds approximately 4 times roll damping ratio $\mu=\delta/\omega_\phi$; then the occurrence of parametric rolling is possible.

The variation of $\overline{GM}$ can be modelled with a cosine or sine function; the equation of roll motion is expressed as follows:

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_{\phi m}^2[1 - h\cdot\cos(\omega_e t)]\phi = 0$$

where

ω_e encounter frequency and

ε an appropriate phase difference.

The value of frequency $\omega_{\phi m}$ here could be slightly different from the natural frequency in calm water, since it is expressed as:

$$\omega_{\phi m} = \sqrt{\frac{W\overline{GM}_{\mathrm{m}}}{I_x + A_{44}}}$$

as the mean metacentric height in wave $\overline{GM}_{\mathrm{m}}$ may be different from the calm water $\overline{GM}$

In the vicinity of exact principal resonance, the following expression may be used for the threshold level of the scaled $\overline{GM}$ fluctuation (Francescutto *et al.* (2004); Spyrou, (2005)):

$$h = \sqrt{\left(2 - \frac{\omega_e^2}{2\omega_{\phi m}^2}\right)^2 + 4\cdot\mu_m^2\frac{\omega_e^2}{\omega_{\phi m}^2}}$$

where, damping ratio is $\mu_m=\delta/\omega_{\phi m}$ is expressed in terms of frequency $\omega_{\phi m}$.

Increase of value h leads to quicker the build-up of parametric rolling.

3.3 Level 1 Prediction of Amplitude of Parametric Roll

If the probability of parametric rolling is "controlled" yet not completely eliminated by design, it is essential to ensure that the amplitude of parametric rolling oscillation that might be generated in an extreme seaway is kept small. For a typical ship with nonlinear restoring and damping, parametric rolling is usually bounded, reaching a steady-state amplitude that is proportional to the square root of the amplitude of restoring fluctuation. In this section, nonlinear formula of restoring moment first, and nonlinear formula of damping are shown.

3.3.1 Closed-form Formulae for Magnitude of Parametric Roll

More specifically, if the amplitude of roll is small to moderate, and depending on the detailed shape of the restoring lever, a third-order polynomial could reasonably represent the exact shape of the initial part of the lever $\overline{GM}_{\mathrm{m}}\phi\cdot(1-c_3\phi^2)$. The nonlinear equation of roll in a longitudinal sea would be like:

$$\ddot{\phi}+2\delta\dot{\phi}+\omega_{\phi m}^2\phi\{[1-h\cdot\cos(\omega_e t)]-c_3\phi^2\}=0$$

Then, the following expression could be used for predicting the steady roll amplitude A in the vicinity of principal resonance (Spyrou, (2005)):

$$A^2=\frac{4}{3c_3}\left[\left(1-\frac{1}{a}\right)\mp\sqrt{\frac{h^2}{4}-\frac{4\mu_m^2}{a}}\right]$$

In the above $a=4\omega_{\phi m}^2/\omega_e^2$, the $\pm$ sign indicates the possibility of multiple, stable/unstable parametric roll oscillations coexisting for the same frequency ratio. In general, the larger A corresponds to the stable solution which is the realizable amplitude.

If the amplitude of parametric roll is moderate to large, a fifth order polynomial is likely to be required. In such a case the following expression of the amplitude could be useful (Spyrou, (2005)):

$$A^2=-\frac{3c_3}{5c_5}\mp\sqrt{\left(\frac{3c_3}{5c_5}\right)^2-\frac{8}{5c_5}\left(-1+\frac{1}{a}\mp\sqrt{\frac{h^2}{4}-\frac{4\mu_m^2}{a}}\right)}$$

In the above, c_3, c_5 are nonlinear stiffness coefficients, corresponding respectively to the third and fifth order restoring terms, according to the following roll equation:

$$\ddot{\phi}+2\delta\dot{\phi}+\omega_{\phi m}^2\phi\{[1-h\cdot\cos(\omega_e t)]-c_3\phi^2-c_5\phi^4\}=0$$

The given formula for the amplitude of parametric roll can be used if the restoring curve is initially hardening (c_3<0) and then softening (c_5>0). It can be deduced that up to 4 coexisting stable/unstable solutions become possible for some values of the frequency ratio. For $a\geq 1$, the solution of the smallest amplitude is stable and stability alternates as we move towards the coexisting higher roll amplitudes, for the same value of the frequency ratio a. For $a<1$ the principle is the same, however we should start with an unstable solution.

In usual case, roll damping is also nonlinear, and linear and cubic damping coefficients are used. By including a cubic damping term $\delta_3\dot{\phi}^3$ the equation of amplitude of roll motion with a third-order restoring term is modified.

$$\ddot{\phi}+2\delta\dot{\phi}+\delta_3\dot{\phi}^3+\omega_{\phi m}^2\phi\{[1-h\cdot\cos(\omega_e t)]-c_3\phi^2\}=0$$

The steady amplitude should become

$$A^2=\frac{4}{3c_3}\left[\left(1-\frac{1}{a}\right)\mp\sqrt{\frac{h^2}{4}-\left(\frac{4\mu_m}{\sqrt{a}}+\frac{4\mu_{3m}}{a^{3/2}}\right)^2}\right]$$

where cubic damping ratio is $\mu_{3m}=\delta_3/\omega_{\phi m}$.

The nonlinear damping is much smaller than the coefficient of the linear. Quantitative assessment of the various contributions to the amplitude A suggested that the effect of nonlinear damping to the reduction of *A* is much lower than that of the linear (Spyrou, 2005). Parametric rolling could be characterised as "severe" if the steady amplitude is higher than 15 deg. The above simple expressions can be used in order to check whether this limit is exceeded. The accuracy of these expressions was confirmed through comparison with numerical solutions of the corresponding roll equations (see Spyrou et al. 2008).

3.3.2 Semi-analytical and Continuation Methods for Prediction of Magnitude of Parametric Roll

In case the righting arm curve has more complex form, there may be more stable and unstable solutions. To all these solutions and formally determine their stability status methods of analytical mechanics, such as the harmonic balance, the averaging and the multiple scales method can be applied. However, a semi-analytical context is necessary since, usually, closed form expressions of roll amplitude cannot be deduced.

The averaging method, for example, approximates a solution with a cosine or sine function with frequency of principal parametric resonance, time-dependent amplitude and phase shift. The results are usually computed from a nonlinear algebraic equation for amplitude. While it requires more computational efforts compared to closed-form formulae, it has fewer limitations in term of the shape of the righting arm and models of roll damping.

The averaging method solution for the case of n-power polynomial presentation of the $\overline{GZ}$ curve and cubic damping is available from (Hashimoto et al., 2004) and (Maki et al. 2011). Formulae for stability analysis are available from (Maki et al. 2011). Sakai et al. (2017) extends the averaging method for the case of numerical representation of the calm-water $\overline{GZ}$ curve (i.e. no approximation is necessary). The essential results are also available from Annexes 1 and 2 of SDC 5/6/2.

In fully numerical context, the continuation method, is recommended for the efficient identification of parametric roll steady states as some systems parameters (such as the speed or the wave height) are varied. This technique is applicable also for multi-degree of freedom mathematical models and hence it can be interfaced with the more complex mathematical models of levels 2 and 3 described in sections 4 and 5. The continuation method is based on the "path-following" technique, where the curve of steady-states is traced directly without performing simulation. Usually a predictor-corrector algorithm is employed. This allows finding the unstable solutions through the procedure applied also for the stable. Moreover, bifurcated solutions (e.g. due to a period-doubling phenomenon) can also be traced Example of application of continuation method to evaluate the magnitude of parametric roll can be found, for example, in (Spyrou and Tigkas 2007, Spyrou et al. 2008).

4. PREDICTION OF OCCURRENCE AND MAGNITUDE OF PARAMETRIC ROLLING

4.1 Using Time-domain Simulation for Prediction

While level 1 is focused on "quick" calculation that can be done without professional software, the level 2 prediction of occurrence and magnitude is based on simplified (compared to potential-flow codes) time-domain simulation of ship motions in waves. Time-domain simulation is numerical integration of equation of motions describing parametric roll. For the level 2 prediction, time domain simulation is carried out for a series of regular waves with given amplitude and frequencies.

The expected result is a set of response curves computed for different wave amplitudes and ship speeds. Each response curve shows dependency of magnitude on the wave frequency. Wave heading for level 2 is limited to following and head seas. The calculations are usually repeated for several loading conditions.

Two types of mathematical models can be used for the level 2 prediction. One uses pre-calculated stability-in-waves (see section 4.2). Another one computes Froude-Krylov and hydrostatic forces and moments using pressure integration over the submerged part of the hull or evaluates an instantaneous submerged volume and it geometric centre, see sections 4.3 and 4.4.

Level 2 prediction requires ship hull geometry to be available. The ship geometry is used to compute stability curves in waves with specialized software or used directly for computation of Froude-Krylov and hydrostatic forces and moment. The geometry may be limited by water-tight volumes or include weather-tight volumes as well. The justification for including weather-tight volume is that the large roll angle during parametric roll does not last long enough for water to penetrate into the weather-tight volume.

Inclusion or non-inclusion of weather-tight volume is a matter of convenience in the most practical cases. The difference can only be seen when a deck enters water, so the parametric roll can already be characterized as “severe”. This difference, however, may have practical significance for low freeboard ships, where inclusion of weather-tight volume may be a matter of realistic modelling. Thus, prediction of occurrence and magnitude of parametric roll for low freeboard ships requires caution.

When the parametric roll is severe and only water-tight volume is included (usually), the simulation may indicate capsizing by showing transition to another stable equilibrium (180 degrees) or exceedance of a large angle (90 degrees). While these cases do not necessarily mean that capsizing will occur, they should be taken as an indication of extremely severe parametric roll.

Presence of parametric roll is usually indicated by stabilization of a single-amplitude solution after some transition period. Magnitude of parametric roll is found as average of last 8 zero-crossing maxima and minima in the time history.

If roll amplitude is not stabilizing, but continues to grow, the wave frequency is close to the boundary of the range of occurrence of parametric roll. To adjudicate such a case, first the length of the time history can be increased twice; if the roll motions still do not stabilize, then the wave frequency should be increased or decreased, until the definite results is obtained.

To shorten the transition period, it is recommended to set initial roll value to 5 degrees. All other initial condition may remain at 0. Length of the time history should be not less than 30 natural roll periods.

Absence of parametric roll is established by observing decaying roll oscillations.

Frequency range to search for parametric usually covers from $1.75\omega_\phi$ to $2.25\,\omega_\phi$, but should be extended if disappearance of parametric roll was not observed with both boundaries of the range.

4.2 Single DoF Mathematical Models for Level 2 Prediction

Singe DoF mathematical model for prediction of parametric roll uses pre-calculated stability curves in regular waves for a given amplitude a_w and frequency of the wave ω_w being a function of roll angle and position of the midship section relative to the wave crest *x*, index "*w*" identifies wave parameters or a value dependent of wave parameters:

$$\overline{GZ}_W(\phi, x) = \overline{GZ}_W(\phi, x; a_w,\ \omega_w)$$

The stability curve in waves is computed with appropriate specialized software with full balancing in heave and pitch on a curved water surface. That means that for each heel angle and position on the wave, the ship is sunk and trimmed to achieve equilibrium in heave and pitch. The result is usually presented in a form of a table by heel angles and wave crest position, for a given wave, while the instantaneous value is computed with linear interpolation.

Mathematical model for parametric roll is described by the ordinary differential equation, (similar to the one, introduced in subsection 3.1):

$$(I_x + A_{44})\ddot{\phi} + B_\phi(\dot{\phi}) + W\overline{GZ}_W(\phi, x) = 0$$

Where roll damping moment is expressed as:

$$B_\phi(\dot{\phi}) = B_{\phi 1}\dot{\phi} + B_{\phi 2}\dot{\phi}|\dot{\phi}| + B_{\phi 3}\dot{\phi}^3$$

The meaning and methods of computation of these coefficients are discussed in the procedure 7.5-02-07-04.5 "Numerical Estimation of Roll Damping". Instantaneous position of the ship relative to the wave crest is computed as:

$$x = (c_w - v_s)t$$

Where c_w is the wave celerity, v_s is ship speed and *t* is time

For the prediction of parametric roll in following seas, the performance of a single-degree-of-freedom mathematical model is expected to be reasonable; as pitch and heave are not very large in following seas. Hashimoto et al.(2004) reported some overestimation of parametric roll magnitude if the calculations are based on the Froude-Krylov assumption only.

4.3 Three DOF Mathematical Models for Level 2 Prediction

For the prediction of parametric roll in head seas, heave and pitch motions should be considered because they are coupled with the roll motion and cross-coupling radiation forces are induced when the roll angle is not zero. Use of a coupled model of roll, heave and pitch is recommended for dynamic analysis of parametric roll in head seas, particularly when ship speed is not zero.

Pre-calculation of the stability curve in waves for 3 DoF, while possible in not practical, as it requires handling a volume of data that is too large. Instead, restoring and excitation in wave can be computed by integration of hydrostatic and wave pressures over the instantaneous submerged part of the hull.

Computational procedure for pressure integration is mature and well tested. Weems et al. (2018) describes a fast algorithm for computation of Froude-Krylov and hydrostatic forces by computing an instantaneous volume and its geometric centre as an equivalent to pressure integration.

The coupled equations of motion are expressed as:

$$\begin{cases} (m + A_{33})\ddot{\zeta} + B_{33}\dot{\zeta} + F_z^{FKHS}(\zeta, \phi, \theta, t) = 0 \\ (I_x + A_{44})\ddot{\phi} + B_\phi(\dot{\phi}) + M_x^{FKHS}(\zeta, \phi, \theta, t) = 0 \\ (I_y + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + M_y^{FKHS}(\zeta, \phi, \theta, t) = 0 \end{cases}$$

Where ζ is heave displacement, θ is pitch angle, *m* is mass of the ship, I_y is moment of inertia in pitch, A_{33} and A_{55} are added masses in heave and pitch motion respectively, B_{33} and B_{55} are damping coefficients in heave and pitch respectively.

This formulation, in principle, allows consideration of any heading to waves as well as consideration of the irregular waves. However, consideration of these more complex problems calls for application of potential flow /hybrid codes addressed in section 5.

4.4 Six DOF Mathematical Models for Level 2 Prediction

The model described in the subsection 4.3 has been further extended to 6 degrees of freedom using manoeuvring coefficients (Weems et al 2018). The model allows consideration of the influence of surging on parametric roll that may be important (Spyrou, 2000).

5. PREDICTION OF OCCURRENCE AND MAGNITUDE OF PARAMETRIC ROLLING

5.1 Application of Hybrid / Potential Flow Hydrodynamic Codes for Prediction in Regular Waves

Several numerical models for parametric rolling were developed and some of them were validated with their model experiments in head and following waves (Reed, 2019). These models are mostly based on coupled heave-pitch-roll models using simultaneous nonlinear differential equations and the hydrodynamic coefficients used in the equations are calculated with potential theories and empirical viscous force estimation. Nowadays CFD (Computational Fluid Dynamics) calculation could be an alternative in the estimation of roll damping coefficients (e.g. Report of the Stability in Wave Committee, 2014). Time variation of hydrodynamic coefficients of radiation and diffraction forces is recommended to include because they change significantly when large amplitude parametric roll happens. The nonlinear radiation and diffraction forces, as well as the nonlinear Froude-Krylov force, are important elements for the prediction of parametric roll in head seas.

Prediction of parametric roll in head and following seas can be successfully performed by potential-flow codes, providing reasonable comparison with the model test (France et al., 2003). The comparison can be improved by avoiding duplication in roll damping as described in subsection 5.1.2.

For the prediction of parametric roll in oblique seas, above-mentioned heave-pitch-roll coupling motion models may not be sufficient, because manoeuvring motion including rudder actions are unavoidable in this situation. Lin et al. (2006) demonstrated how duplication can be avoided for manoeuvring forces (subsection 5.3.1).

Umeda et al. (2015) attempted to validate a 5 degrees-of-freedom (sway-heave-pitch-roll-yaw) numerical simulation, taking low-speed manoeuvring model, in oblique seas by comparing with measured results conducted in a seakeeping and manoeuvring model basin. The numerical model, including the nonlinear Froude-Krylov force, radiation and diffraction forces calculated as a function of roll angle and manoeuvring forces, can predict experimental results qualitatively. Further improvement is expected for quantitative prediction of parametric roll in quartering seas.

5.1.1 Requirements for the Hydrodynamic Code

The level-3 computational tool typically includes:

- Body-nonlinear formulation (i.e. computed on instantaneous submerged part) for Froude-Krylov and hydrostatic forces;
- Body-nonlinear or body linear (i.e. computed over averaged waterplane) potential flow formulation for diffraction and radiation forces

- Polynomial models for viscous-related forces for, propulsors, appendages, manoeuvring and roll damping. Coefficients in these polynomials are estimated by empirical formulae, model test of CFD calculations.
- 3 DOF to 6 DOF dynamic solvers. For 6 DOF, directional control should be modelled or simulation of elastic strings (to keep direction) should be available

5.1.2 Avoiding Duplication in Roll Damping

Roll damping includes viscous and wave components. Polynomial models of roll damping based on empirical formulae, model test or free-surface CFD calculation include all components of roll damping. Radiation forces computed with potential flow calculation also include wave forces. Using polynomial models of roll damping together with potential flow calculation lead to duplication of the wave forces.

To avoid duplication, roll damping polynomial coefficients should be calibrated to match roll decay test carried out by the advanced code. Following the procedure 7.5-02-07-04.5, cubic or quadratic polynomial is fitted to the decay curve obtained form numerical simulation of roll decay:

$$\Delta\varphi = a\phi_m + b\phi_m^2 + c\phi_m^3$$

where $\Delta\phi = \phi_{n-1} - \phi_n$, $\phi_m = 0.5(\phi_{n-1} + \phi_n)$, ϕ_n is an absolute n-th extreme value roll decay time history, while a, b and c are decay coefficients. The fitting can be presented as vector valued function:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \vec{F}(\{\phi_n\}); \;\; n = 1, \dots, N$$

where N is a number of extreme values in the roll decay time history. Given a_0,b_0, c_0 are the described values of the decay coefficients, corresponding to the results roll decay test of a numerical method described in the procedure 7.5-02-07-04.5, the calibrated roll decay coefficients is computed from the following algebraic equation, using any appropriate numerical method

$$\begin{pmatrix} a_0 \\ b_0 \\ c_0 \end{pmatrix} - \vec{F}\left(\{\phi_n\}_j\right) = 0$$

where j is j-th iteration of the chosen numerical method leading to acceptable tolerance of the solution. Finally the calibrated roll decay coefficients a_c, b_c, c_c are expressed as:

$$\begin{pmatrix} a_c \\ b_c \\ c_c \end{pmatrix} = \vec{F}(\{\phi_n\})$$

5.1.3 Avoiding Duplication in Manoeuvring Forces

"Manoeuvring" force of a viscous nature may also include a wave component if measured from a model test in calm water or computed with free-surface CFD. Coefficient of "manoeuvring" forces in this case should be calibrated in a manner, similar to described in subsection 5.1.2, see Lin et al (2006).

5.1.4 Choice of Conditions and Presentation of the Results

Choice of conditions and form of presentation of the results of numerical prediction of magnitude and occurrence of parametric roll on regular waves are analogous between level 2 and level 3. Section 4.1 is fully applicable for the level 3 assessment.

5.2 Application of Hybrid / Potential Flow Hydrodynamic Codes for Prediction in Irregular Waves

5.2.1 Specific Properties of Parametric Roll in Irregular Waves

The variation of $\overline{GM}$ that is theoretically tolerable for a regular wave environment, in the sense that it does not give rise to parametric rolling, should be distinguished from the $\overline{GM}$ variation that could be practically permissible in a realistic seaway. The limited (rather than infinite) run length of critical wave groups and the possibly low probability to be encountered by a ship, mean that if the standard approach based on the deterministic criterion of asymptotic stability is applied, the ensuing design requirements may become unnecessarily stringent.

The narrower the sea spectrum, the more prominent becomes the wave groupness, and the higher the probability of exceeding the threshold. In a following sea, the $\overline{GM}$ fluctuation could show, for an observer moving with the ship, a very narrow spectrum, even if the sea spectrum is quite wide (e.g. Bretschneider spectrum). This can result in a concentration of wave energy within a very narrow range of encounter frequencies, for certain heading angles in following/ quartering seas. Subsequently, a ship could experience a dangerous, regular-like parametric excitation if the frequency condition associated with parametric roll is approximately satisfied and if the associated waves are of critical height and length. On the other hand, in a head sea, the $\overline{GM}$ fluctuation could show a wide spectrum even in moderately narrow spectra (e.g. JONSWAP). Model tests have shown that parametric rolling can be excited even in head seas very quickly during the passage of a wave group with critical characteristics.

Questions have been raised recently about the assumption of practical ergodicity of parametric roll during model testing in a wave basin and for numerical simulations of finite duration. This could create some uncertainty for current experimental or numerical assessment methods if these are based on finite temporal averages of roll motion. Reed (2019) recommends using at least 20 half-hour records for reliable estimation

5.2.2 Choice of Conditions

Magnitude of parametric roll is evaluated for a particular sea condition characterized by a specific spectral density and its parameters. In the case of two-parameter spectrum, its significant wave height and mean zero-crossing period or modal period.

Irregular waves are assumed long crested for the sake of conservatism

Operation parameters include speed and wave heading. For complete analysis of parametric magnitude at a particular sea state, speed increment of 5 knots and wave heading increment of 15 degrees are recommended.

5.2.3 Processing and Presentation of the Results

Magnitude of parametric roll in irregular waves is a random variable. Its value is judged by a statistical estimate as it is done in any other case of ship motions in irregular waves. An estimate of single significant amplitude (SSA) is conventionally used for this purpose. Procedure 7.5-02-01-08 “Single Significant Amplitude and Confidence Intervals for Stochastic Process: contains detail description of estimation of SSA.

Application of this procedure to parametric roll data carries the following specific features:

- Parametric roll is not a normal stochastic process (Belenky et al. 2011, Hashimoto et al. 2011, Mohamad and Sapsis, 2016) and the SSA should be estimated through direct counting as described by formula (10) in Procedure 7.5-02-01-08, i.e. as an average of 1/3 largest peaks.
- Parametric roll is usually characterized by relatively long time duration for reaching independence.

A problem of capsizing observation should be address separately:

- Capsizing may be observed as an exceedance of a maximum angle specific to a hydrodynamic code (say 90 degrees) or as a transition to roll motion at an "upside-down" stable equilibrium in roll
- If hull geometry includes only water-tight volumes, observed capsizing is a result of non-inclusion of the weather-tight volumes that, in fact, affects Froude-Krylov and hydrostatic forces.
- Application of procedure 7.5-02-01-08 to dataset containing capsizing is possible, but may lead to slightly lower values of SSA.
- Including of weather-tight volume into hull geometry model is recommended if severe parametric roll may be expected.
- Observation of capsizing when hull geometry includes water-tight volumes only is an indication of severe roll response. A capsizing, observed with the weather-tight volume included, has to be taken as an indication of actual problem with dynamic stability.

6. LIST OF SYMBOLS

A	Magnitude of parametric roll, rad
a_w	Amplitude of wave, m
A_{33}	Added mass in heave, kg
A_{44}	Added mass in roll, kg m^2
A_{55}	Added mass in pitch, kg m^2
B_{33}	Damping coefficient of heave, kg /s
B_{55}	Damping coefficient of pitch, kg m^2/s
B_ϕ	Roll damping (function of roll rate) Nm
$c_{3,5}$	Polynomial coefficients of approximation of calm-water $\overline{GZ}$ curve, m
c_W	Wave celerity, m/s
F_z^{FKHS}	Froude-Krylov and hydrostatic force in heave, N
m	Mass displacement of a ship, kg
M_x^{FKHS}	Froude-Krylov and hydrostatic moment in roll, Nm
M_y^{FKHS}	Froude-Krylov and hydrostatic moment in pitch, Nm
$\overline{GM}_{\max}$	Maximum $\overline{GM}$ during the wave pass
$\overline{GM}_{\min}$	Minimum $\overline{GM}$ during the wave pass
$\overline{GM}_{\mathrm{m}}$	Mean $\overline{GM}$ during the wave pass
$\overline{GM}_{\mathrm{m}}$	Mean $\overline{GM}$ during the wave pass
$\overline{GZ}_W$	Stability arm during the wave pass
I_x	Mass moment of inertia in roll, kg m^2
I_y	Mass moment of inertia in pitch, kg m^2
h	Magnitude of parametric excitation, s^{-2}
v_s	Ship forward speed, m/s
W	Weight displacement, N
x	Position along the hull of a ship, m
δ	Dimensional coefficient of linear roll damping, s^{-1}
δ_3	Dimensional coefficient of cubic roll damping term, s
ζ	Heave displacement, m
θ	Pitch angle, deg
μ	Roll damping as fraction of critical, expressed in term of roll natural frequency ω_ϕ
μ_m	Roll damping as fraction of critical, expressed in term of roll frequency $\omega_{\phi m}$
ϕ	Roll angle, deg or rad
ω_e	Wave encounter frequency, s^{-1}
ω_w	True wave frequency, s^{-1}
ω_ϕ	Natural frequency of roll, s^{-1}
$\omega_{\phi m}$	Frequency of free roll in calm water with metacentric height $\overline{GM}_{\mathrm{m}}$ s^{-1}

7. REFERENCES

ABS (2019) Guide for the Assessment of Parametric Roll Resonance in the Design of Container Carriers, American Bureau of Shipping, Houston, TX, 70 p.

Belenky, V.L., Weems, K.M., W.M. Lin, and J.R. Paulling, 2011, "Probabilistic analysis of roll parametric resonance in head seas", Chapter 31 of "*Contemporary Ideas on Ship Stability*", Neves, M.A.S., Belenky, V., de Kat, J.O., Spyrou, K. and N. Umeda, eds., Springer, ISBN 978-94-007-1481-6, pp. 555-572

France, W. M, M. Levadou, T. W. Treakle, J. R. Paulling, K. Michel & C. Moore (2003). An Investigation of Head-Sea Parametric Rolling and its Influence on Container Lashing Systems. *Marine Tech.*¸ **40**(1):1–19.

Francescutto, A., Bulian, G., Lugni, C., 2004, "Nonlinear and stochastic aspects of parametric rolling". Marine Technology, 41, 2.

Hashimoto, H., Umeda, N., 2004, "Nonlinear analysis of parametric rolling in longitudinal and quartering seas with realistic modeling of roll-restoring moment". Journal of Marine Science and Technology, 9, 117-126.

Hashimoto, H., N. Umeda, and A. Matsuda, 2011, "Experimental Study on Parametric Roll of a Post-Panamax Containership in Short-Crested Irregular Waves" Chapter 14 of "Contemporary Ideas on Ship Stability", Neves, M.A.S., Belenky, V., de Kat, J.O., Spyrou, K. and N. Umeda, eds., Springer, ISBN 978-94-007-1481-6, pp. 267-276.

IMO SDC 7/INF.2 Information collected by the Correspondence Group on Intact Stability (Part A), London, UK, November 2019

IMO SDC 7/WP.6 Report of the Drafting Group on Intact Stability, London, UK, February 2020.

IMO SDC 5/6/2 Comments on the calculation method for parametric roll amplitude in the second check of the level 2 vulnerability criterion for parametric rolling failure mode. Submitted by Japan, London, UK, November 2017.

Maki, A., Umeda, N., S. Shiotani, Kobayashi, E. 2011, "Parametric rolling prediction in irregular seas using combination of deterministic ship dynamics and probabilistic wave theory". Journal of Marine Science and Technology, 16, 294-310.

Mohamad, M.A. and T.S. Sapsis, 2016, "Probabilistic response and rare events in Mathieu's equation under correlated parametric excitation", *Ocean Engineering*, 120:289-297.

Lin, W.M. Zhang, S., Weems, K, Luitt, D. (2006) "Numerical Simulations of Ship Maneuvering in Waves", Proc 26th Symposium on Naval Hydrodynamics Rome, Italy.

Paulling, J. R., S. Kastner & S. Schaffran (1972) Experimental studies of capsizing of intact ships in heavy seas. *US Coast Guard, Technical Report*, 58 p. (Also IMO Doc. STAB/7, 1973)

Paulling J. R. & R. M. Rosenberg (1959) On Unstable Ship Motions Resulting from Nonlinear Coupling. *J. Ship Res.*, **3**(1):36–46.

Reed, A.M. (2019) "26th ITTC Parametric Roll Benchmark Study" Chapter 37 of *Contemporary Ideas on Ship Stability. Risk of Capsizing*, Belenky, V., Spyrou, K., van Walree F., Neves, M.A.S., and N. Umeda, *eds*., Springer, ISBN 978-3-030-00514-6, pp. 619-636.

Report of the Stability in Wave Committee, 2014, "9.2.3 CFD-based Prediction of Roll Damping" Proceedings of the 27th ITTC, 397-399.

Sakai, M., Umeda, N., Yano, T., Maki, A., Yamashita, Y., Matsuda, A. and D. Terada (2018) "Averaging methods for estimating parametric roll in longitudinal and oblique waves", *J. Mar Sci Technol.*, **23**: 413–424 doi.org/10.1007/s00773-017-0490-6

Shin, Y.S, Belenky, V.L., Paulling, J.R., Weems, K.M., and W.M. Lin (2004) "Criteria for Parametric Roll of Large Containerships in Longitudinal Seas", *SNAME Trans.* Vol. 112, pp. 14-47.

Spyrou, K.J., 2000, "On the parametric rolling of ships in a following seas under simultaneous periodic surging", *Phil. Trans. R. Soc. Lond. A,* 358:1813-1834.

Spyrou, K.J., 2005, "Design criteria for parametric rolling". Oceanic Engineering International, ECOR, 9, 1.

Spyrou, K.J., Tigkas, I., 2007, "Principle and application for continuation methods for ship design and operability analysis" *Proc. Of 10th Intl. Symp. on Practical Design of Ships and Other Floating Structures PRADS'2007*, Houston. Texas, USA, Vol.1, pp 388-395.

Spyrou, K.J., Tigkas, I., Scanferla, G., Pallikaropoullos, N. and N. Themelis, 2008, "Prediction potential of the parametric rolling behaviour of a post-panamax containership", *Ocean Engineering*, **35**, 1235-1244.

Umeda, N., Fujita, N., Morimoto, A., Sakai, M., Terada, D., Matsuda, A., 2015, "Numerical Prediction of Parametric Roll Resonance in Oblique Waves". Proceedings of the 12th International Conference on the Stability of Ships and Ocean Vehicles, 331-339.

Appendix A. CALCULATION EXAMPLE

A.1. Input Data

The example in this appendix is preformed for C11-class containership. Principle dimensions and loading conditions data are placed in Table A1. All calculations for parametric roll are carried out for zero forward speed.

Superstructure and containers are considered to be weather-tight volumes and are modelled as shown in Figure A1. Calm-water $\overline{GZ}$ curves are shown in Figure A2 with and without superstructure and containers included.

Table A1 Principle Input Data

Length BP, m	262
Beam, m	40
Levels 1, 2 and 3 (Regular waves)	
Mean draft, m	12.7
Trim, deg	0
KG, m	19
GM, m	1.29
Natural frequency of roll ω, 1/s	0.199
Level 3 (Irregular waves)	
Mean draft, m	11.5
Trim, deg	0
KG, m	18.95
GM, m	1.4

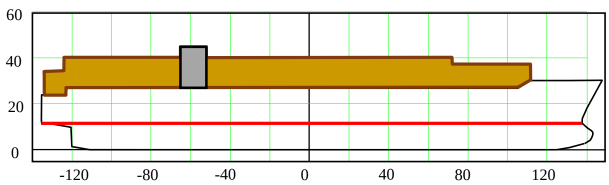

Figure A1 Superstructure and containers as a weather-tight volume

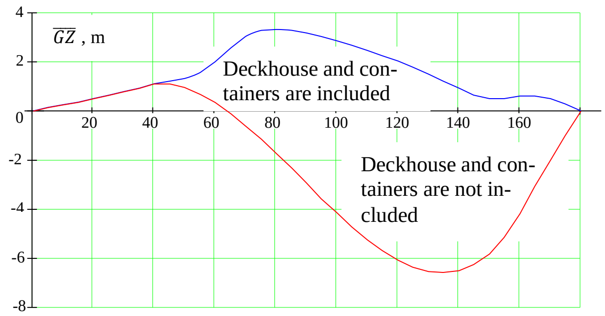

Figure A2 Calm-water righting curves

7.1 Results of Level 1 Prediction

For the level 1 prediction of occurrence of parametric roll values of $\overline{GM}$ in waves were computed for the range of wave frequencies from 0.3 rad/s to 0.5 rad/s (wave lengths from 246.5 m to 684.7 m) and wave height of 2 m. The values of $\overline{GM}$ in waves are shown as functions of wave crest position for all the wave frequencies/lengths in Figure A3. Following recommendations in paragraph 2.3.2.1 of Annex 3 of SDC6/WP.6, ship was balanced in buoyancy and trim at each position of wave crest.

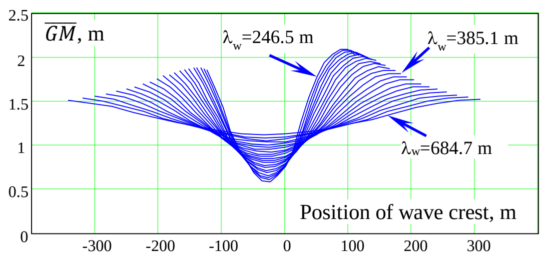

Figure A3 $\overline{GM}$ values in waves: wave height 2 m wave circular frequencies from 0.3 s^{-1} to 0.5 s^{-1}

Occurrence of the parametric roll is analysed as described in subsection 3.2. As zero forward speed is considered the encounter frequency equals the wave frequency. The results are shown in Figure A4. Occurrence of the parametric roll is expected in the between the wave frequencies 0.37 rad/s and 0.44 rad/s, where the amplitude of parametric excitation h is below the threshold defined by the formula in the subsection 3.2.

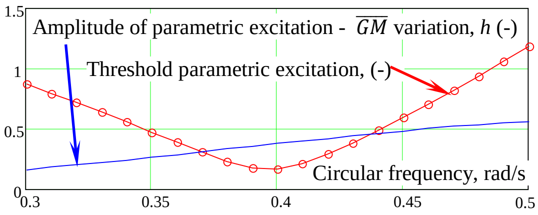

Figure A4 Occurrence of the parametric roll. Roll damping in term of critical μ_m=0.04, wave height 2 m

To assess magnitude of parametric roll, calm-water $\overline{GZ}$ has to be approximated by cubic or 5th-order polynomial. The calculations are done with “water-tight” $\overline{GZ}$ as it is ususally readily available (expected use of the level 1 assessment is express analysis that can be performed without specialized software.) Polynomials were fitted with method of least squares. The number of heel angles is limited to approximately match position of the maximum of actual $\overline{GZ}$ curve. Results of the fitting are shown in Figure A5.

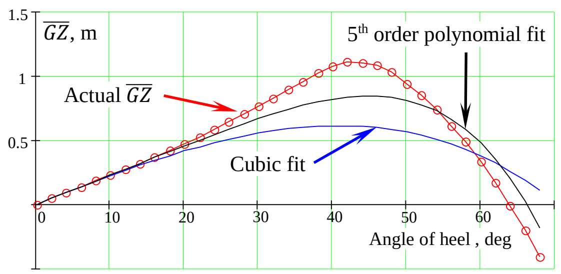

Figure A5 Fitting calm-water $\overline{GZ}$ curves. Cubic fit is done up to 70 degrees, 5th order fit – up to 80 degrees

Magnitudes of parametric roll (in a form of response curves) are shown in Figure A6 for linear roll damping and in Figure A7 for linear-plus-cubic roll damping.

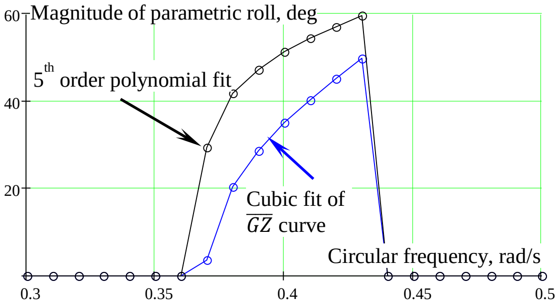

Figure A6 Magnitude of parametric roll, wave height 2 m, roll damping μ_m =0.04

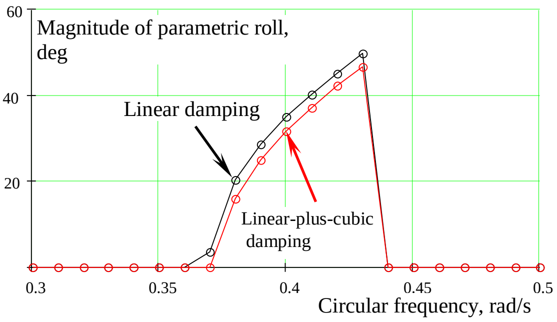

Figure A7 Magnitude of parametric roll, wave height 2 m, roll damping: linear μ_m=0.04, linear-plus-cubic μ_m=0.018 μ_{3m} =0.6

A.2. Results of Level 2 Prediction

Prediction of parametric roll magnitude with numerical solution of single degree-of-freedom roll equation is described in subsection 4.2. It requires $\overline{GZ}$ curves in waves computed for the entire range of wave lengths included in the analysis. These calculations were done for 40 circular wave frequencies ranging for 0.3 to 0.5. Firstly, containers and superstructure were not included, i.e. calculations were done for water-tight volume. An example of $\overline{GZ}$ curves in waves is shown in Figure A8, while time history of roll motions is placed in Figure A9.

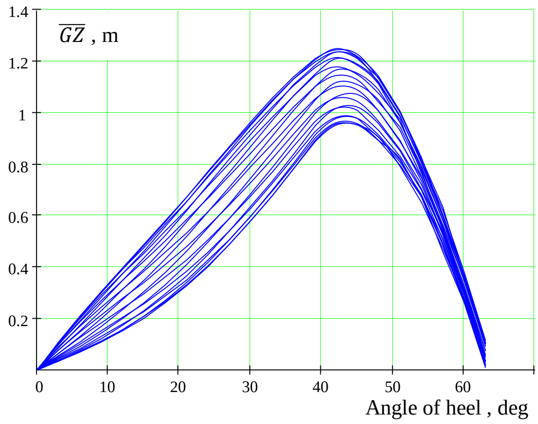

Figure A8 $\overline{GZ}$ curves in waves for water-tight volume, wave height 2 m, wave length 262 m, circular wave frequency 0.485 s^{-1}

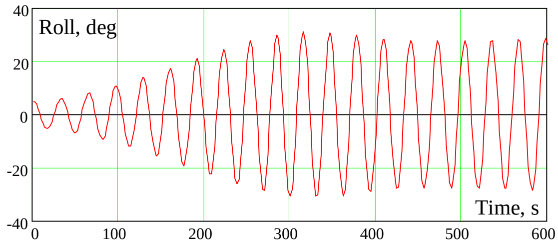

Figure A9 Time history of roll, initial angle 5 degrees, wave height 2 m, wave length 385.1 m, circular wave frequency 0.4 s^{-1}

Inclusion of the superstructure and containers (i.e. consideration of the weather-tight volume) leads to the significant change of the $\overline{GZ}$ curves in waves, see an example in Figure A10.

Figure A11 shows response curves for magnitude of parametric roll computed for water-tight and weather-tight volumes. The difference between the curves can be attributed to hardening nonlinearity caused by inclusion of the superstructure and containers while computing $\overline{GZ}$ curves in waves.

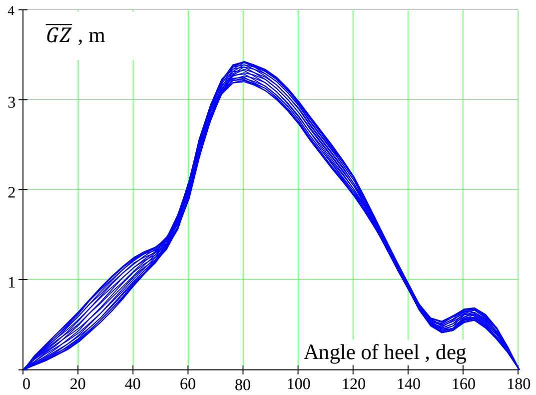

Figure A10 $\overline{GZ}$ curves in waves for weather-tight volume, wave height 2 m, wave length 262 m, circular wave frequency 0.485 s^{-1}

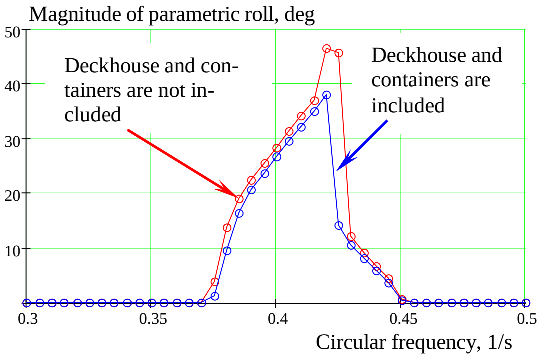

Figure A11 Magnitude of parametric roll by numerical integration of single DOF roll equation, wave height 2 m, roll damping μ_m =0.04

As mentioned in subsection 4.1, another aspect of using water-tight volume is indication of capsizing, as the stable steady state cannot be established for given initial conditions or does exist for given nonlinearity and parametric excitation. Example of capsizing time history is shown in Figure A12, while the corresponding response curves for magnitude of parametric roll are shown in Figure A13.

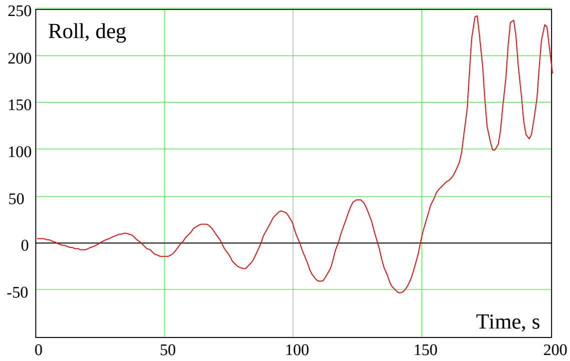

Figure A12 Time history of roll, initial angle 5 degrees, wave height 4 m, wave length 366.6 m, circular wave frequency 0.41 rad/s

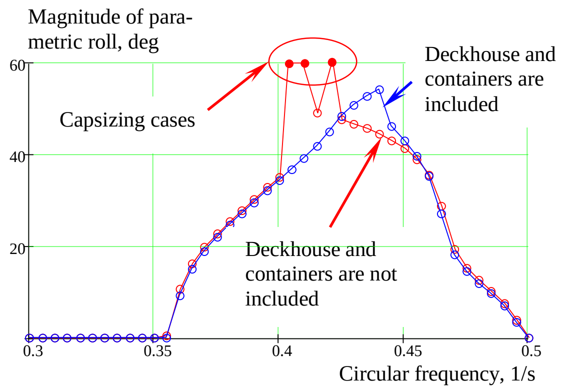

Figure A13 Magnitude of parametric roll by numerical integration of single DOF roll equation, wave height 4 m, roll damping μ_m =0.04

As recommended in subsection 4.3, 3-DOF calculations of parametric roll were carried out using volume-based algorithm that is equivalent to wave and hydrostatic pressure calculation on an instantaneous submerged portion of hull. The algorithm is described in (Weems et al 2018).

Calculations were carried out for both water-tight (Figure A14) and weather-tight volumes (Figure A15). Comparing to 1 DOF calculation, one can observe increase both range of occurrence and magnitude of parametric roll.

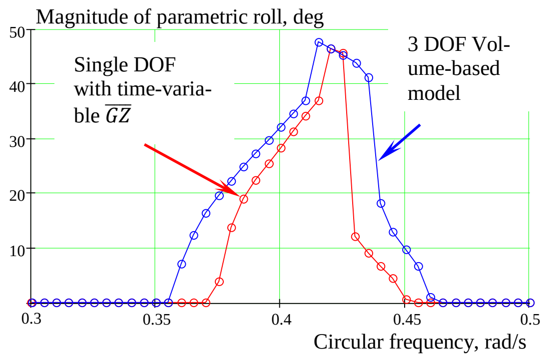

Figure A14 Magnitude of parametric roll. Watertight volume only, wave height 2 m, roll damping μ_m =0.04

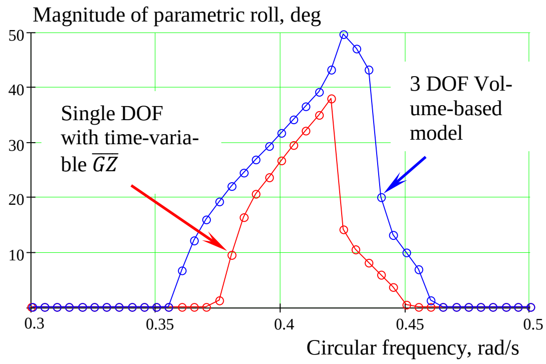

Figure A15 Magnitude of parametric roll. Weathertight volume, wave height 2 m, roll damping μ_m =0.04

7.2 Results of Level 3 Prediction

Large Amplitude Motion Program (LAMP) has been used for the level-3 prediction of magnitude and occurrence of parametric roll. Calculations were done with 3 DOF: heave-roll-pitch. LAMP is a 3D hybrid potential-flow code. Froude-Krylov and hydrostatic forces are computed by pressure integration over the instantaneous portion of submerged hull. The LAMP-2 potential-flow solver was used. The diffraction and radiation forces are evaluated over a mean waterline for a constant forward speed. To asses viscous and lifting forces, including appendages forces coefficient-based computational models are used. More information on previous application of LAMP for parametric roll assessment are available in (France, et al 2003), (Shin, et al 2004) and others.

Geometry of a hull is modelled with 3D panels. Figures A16 and A17 show different views of the hull configuration. Different colours correspond to different surfaces where different interpolation settings can be used. Only watertight volume is modelled.

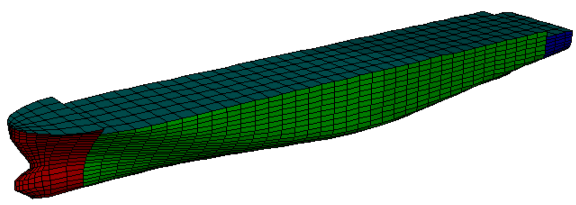

Figure A16 Panel model of a hull: bow view

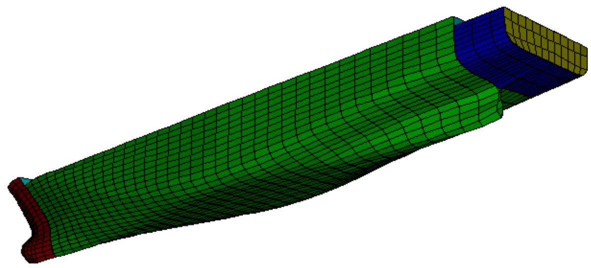

Figure A17 Panel model of a hull: stern view

As it is noted in subsection 5.1.2, wave components of roll damping are computed by the potential-flow solver as a part of radiation forces. Thus inclusion of coefficients evaluated from roll decay test of CFD simulation. The subsection 5.1.2 recommends adjusting roll damping coefficient until the code reproduces the given roll decay data "as tested". Roll decay test data (France, et al 2003), are used – they are shown as points in Figure A18.

The algorithm described in the subsection 5.1.2 was followed in this example. Note that LAMP only uses linear plus quadratic model of roll damping. These coefficients are estimated from Figure A18 as an intersect (linear damping coefficient) and slope (quadratic damping coefficient) of a line fitted to the points obtained from "simulated" roll decay test that are shown as solid lines in Figure A18. Finally calibrated roll decay coefficient are computed from the appropriate equation in the subsection 5.1.2.

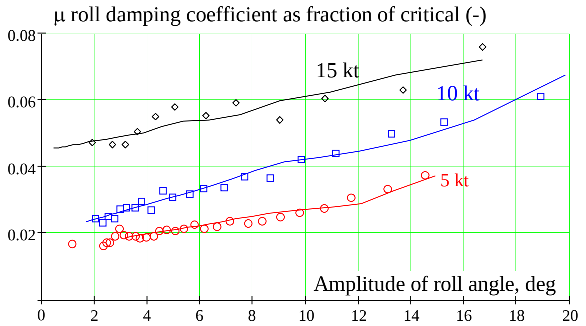

Figure A18 Roll decay test for three speeds (points – taken from France et al 2003) and computed with LAMP (solid lines)

A response curve, computed for zero forward speed is shown in Figure A19. Roll damping coefficients, estimated for the speed of 5 knots were used for the zero forward speed case.

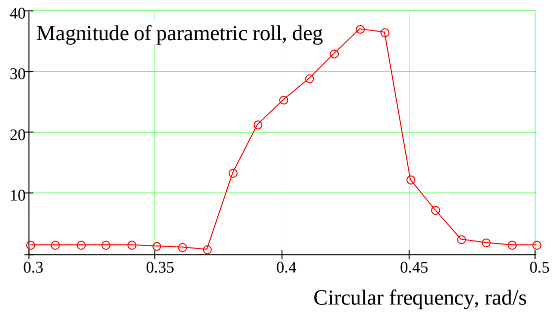

Figure A19 Magnitude of parametric roll computed with LAMP-2, zero forward speed, wave height 2 m.

Comparing the response curve in Figure A19 with the corresponding curves from A6, A7 and A14, a shift of parametric roll occurrence range is observed. It can be explained by the added mass. LAMP-2 computes added mass, while the approximate formula, single DOF and volume-based calculation used usual assumption of the added mass in roll to be 0.25 of the moment of inertia in roll. The shift can be explained by the difference in natural frequency of roll, resulting from the difference in the added mass.

The value of magnitude of parametric roll, calculated by LAMP is less compare to level 1 and level 2 calculations, but still remains "severe". Thus the assessment of magnitude of parametric roll is consistent through all three levels.

Magnitude of parametric roll in irregular wave is a random number. As recommended in section 5.2.3, an estimate SSA is used to assess the magnitude of parametric roll in irregular wave. Since parametric roll is not a normal stochastic process, direct counting is used to estimate SSA as recommended in the Procedure 7.5-02-01-08.

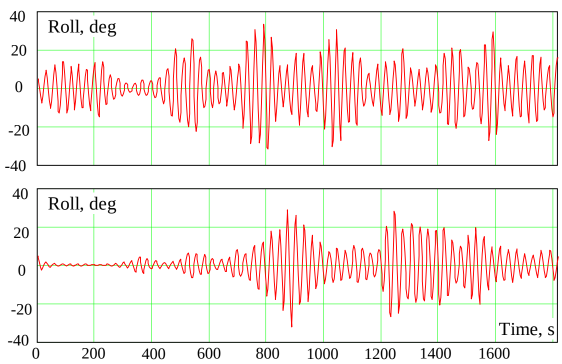

Figure A20 Two sample records of parametric roll simulated for significant wave height 3.5, mean zero-crossing period 14s, heading 1 degree, zero forward speed

As stated in the subsection 5.2.1, assumption of ergodicity is not applicable for parametric roll; SSA is estimated on an ensemble of records. Following recommendations in the subsection 5.2.1, 20 half-hour long records were generated for estimation of SSA. Figure A20 shows 2 of these 20 records.

To estimate the SSA by direct counting, one needs to find an average of 1/3 largest peaks roll motions. Following recommendations in the subsection 3.2.2 of the Procedure 7.5-02-01-08 absolute values of mean-crossing peaks are found. Figure A21 shows the mean-crossing peaks with circles.

To find 1/3rd quantile, the peaks are sorted (highest to lowest) and the value that encompasses the highest 1/3rd peaks is found. The largest 1/3rd peaks are shown in Figure A20 with squares. The SSA is a mean value estimate of the 1/3rd largest peaks over all 20 records (10 hours of simulation time) equals to 22.5 degrees.

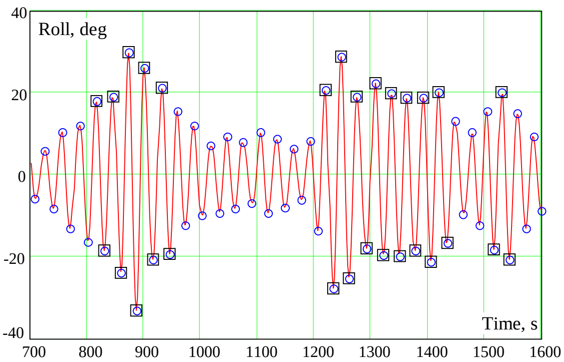

Figure A21 Peaks (circles), 1/3rd largest peaks (squares) shown for a part of the record 1 in Fig. A20.

The subsection 3.2.3 of the Procedure 7.5-02-01-08 contains guidance for assessment of confidence interval of SSA to evaluate statistical uncertainty of the estimate. To address dependence within the simulated motion, a time duration for reaching independence is required. This time is found with an ensemble averaged autocorrelation function of roll motions.

An envelope is fitted to the estimate of the autocorrelation function; see Figure 5 of the Procedure 7.5-02-01-08 and Figure A22 of this document. The level of 0.05 is accepted as a significance level, so correlation below 0.05 is considered insignificant. Thus the time when the envelope of autocorrelation function reaches the level of 0.05 can be taken as "decorrelation time". Assuming that absence of correlation manifests independence, the decorrelation time can be taken as the time duration for reaching independence,

As stated in the subsection 5.2.3 of this document, parametric roll is characterized by a long time for reaching independence - 939 seconds: compare Figure A22 to Figure 5 of the Procedure 7.5-02-01-08, where the time duration for reaching independence for a typical synchronous roll motion is below 50 seconds.

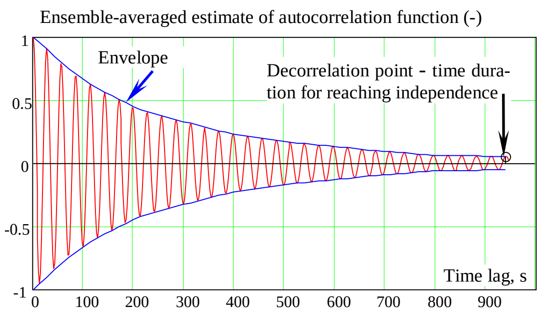

Figure A22 Autocorrelation function and its envelope; time duration for reaching independence 939 s

Following the instructions in the subsection 3.2.3 of the Procedure 7.5-02-01-08, a sample of the largest 1/3rd peaks are separated into independent groups using the time duration for reaching independence of 939 seconds (the peaks occurring further than 939 seconds away of each other are considered independent). In the considered case, the number of independent groups equal to the number of independent records (20), because of the long time to reach independence.

Auto-covariance function of the largest 1/3rd peaks is estimated with formula (12) of the Procedure 7.5-02-01-08. An estimate of auto-correlation function (auto-covariance normalized by variance estimate) is shown in Figure A23. Note, that the estimate of autocorrelation function of the largest 1/3rd peaks is computed for the number / index of the peaks rather than a time lag.

Similar to the autocorrelation function of the instantaneous roll angle values in Figure A22, the envelope is computed and the index to reach independence is determined. It equals for 12 for the considered example.

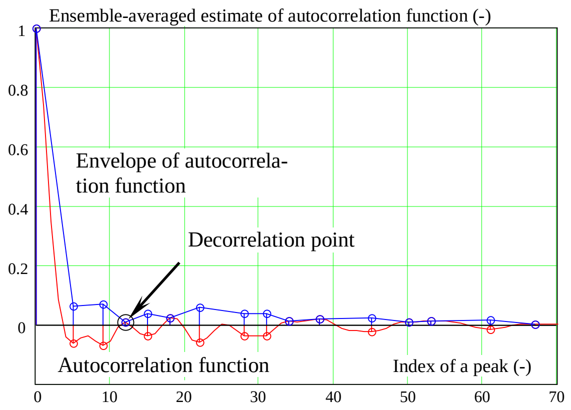

Figure A23 Autocorrelation function and its envelope of of the largest 1/3rd peaks; index / number of peaks for reaching independence is 12

As it is noted in the subsection 3.2.3 of the Procedure 7.5-02-01-08, the autocorrelation function, shown in Figure A23 needs to be “cut” to avoid the influence of inaccuracies in the large indices, caused by the decrease of available data. Decorrelation point is used in these calculations as a “cut-off” point. The result is shown in Figure A24. This “cut-off” function is further used in formula (13) of the Procedure 7.5-02-01-08 to compute the variance of SSA estimate. Note the similarity of Figure A24 with Figure 6 in the Procedure 7.5-02-01-08.

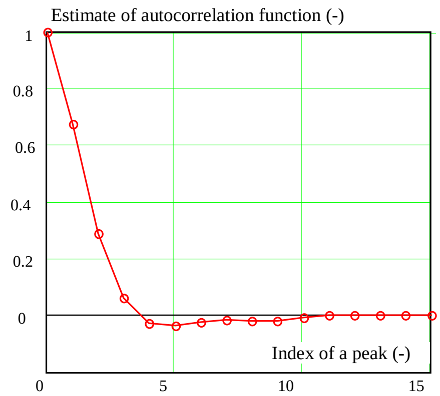

Figure A24 Final estimate for the auto-correlation function for 1/3rd largest peaks of roll motion

The results of SSA estimation are summarized in table A2.

Table A2 SSA Estimation Results

Significant wave height, m	3.5
Mean zero-crossing wave period, s	14
Heading, deg	1
Speed, kn	0
Number of records	20
Duration of a record, min	30
Roll SSA estimate, deg	22.5
Time for roll independence , s	939
Number of independent groups	20
Number of peaks for independence	12
Std. dev. of SSA estimate, deg	0.26
Confidence probability	0.95
Upper boundary, deg	23
Lower boundary, deg	22

Following recommendations in the subsection 5.2.2 of this document, example of handling of capsizing cases is given below. To observe capsizing event the significant wave height has been increased to 9 m, while the mean zero crossing period remains 14s. Speed and heading remain the same. As a result, 19 out of 20 records ended up with the capsizing. Two of these records are shown in Figure A25.

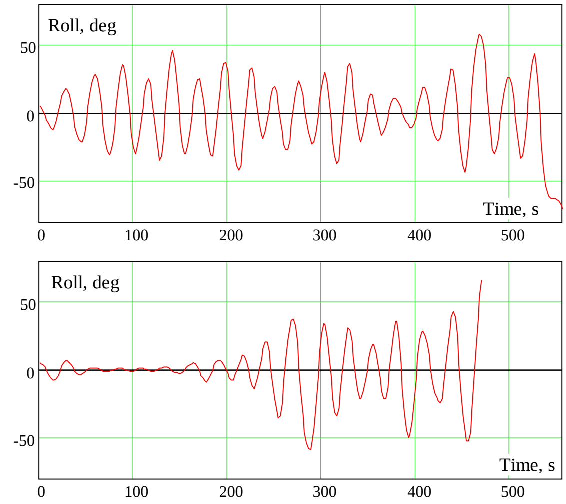

Figure A25 Two sample records of parametric roll with capsizing, simulated for significant wave height 9 m, mean zero-crossing period 14s, heading 1 degree, zero forward speed

As the geometry of the hull does not include weather-tight volumes (containers and superstructure), observation of capsizing is an indication of sever roll response rather than an actual problem of dynamic stability.

The peak search procedure is not influenced by capsizing, see Figure A26, as the definition of a peak presumes “a return”. As capsizing may prevent realization of large-amplitude roll angles, these large peaks are absent from the sample, leading to possible underestimation of SSA. Actual data processing does not differ from the previous case without capsizing cases. The results are summarized in Table A3.

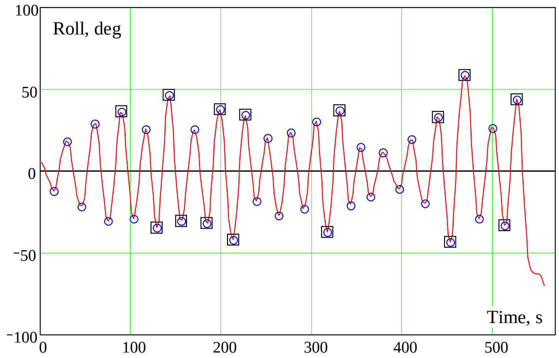

Figure A26 Peaks (circles), 1/3rd largest peaks (squares) shown for a part of the record 1 in Fig. A25.

Table A3 SSA Estimation Results

Significant wave height, m	9
Mean zero-crossing wave period, s	14
Heading, deg	1
Speed, kn	0
Number of records	20
Max duration of a record, min	30
Roll SSA estimate, deg	40.1
Time for roll independence , s	411
Number of independent groups	20
Number of peaks for independence	15
Std. dev. of SSA estimate, deg	0.39
Confidence probability	0.95
Upper boundary, deg	40.9
Lower boundary, deg	39.4