ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

Laboratory Modelling of Multidirectional Irregular Wave Spectra

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Laboratory Modelling of Multidirectional Irregular Wave Spectra

1. PURPOSE OF GUIDELINE

The purpose of this recommended guideline is to ensure that laboratory generated directional waves are modelled and documented according to proper and well defined methods. A practical implementation and use within naval architecture and ocean engineering applications is essential.

It shall also point out particular challenges, limitations and uncertainties inherent in laboratory directional spectrum estimation. Attention is mainly focused at the directional characteristics. Some considerations about the multi-modal power spectra are provided only when they are directly relevant for the directional modelling.

An overview of the most commonly used principles, methods and definitions is given in this procedure, together with some guidelines. It is not the intention to provide particular recipes for all steps in the wave generation and analysis, for which more details can be found in e.g. IAHR (1997).

2. SCOPE

2.1 Use of directional spectra

Real ocean waves are directional (short-crested). However, for practical and simplicity reasons, unidirectional waves have in the past traditionally been modelled in most applications within naval architecture and offshore engineering. Both numerically and experimentally, the generation, analysis and documentation of directional spectra are more complex. Also, the interpretation of model response results may be more challenging. Still, a significant development of experimental facilities and methods has taken place since the 1980's, especially within hydraulic and coastal engineering, and the use is expected to continue to increase. This is also supported by the fact that more ocean field data are becoming available. The goal might be not only to reproduce exact recorded wave data, but also to check the models under complex waves (ULS, fatigue).

The significance of using directional wave modelling, and which characteristics are essential, will depend on the actual application. The normal assumption is that for simple floating bodies, wave loads are generally reduced, while for compliant systems (due to its high nonlinearity), loads may sometimes also increase. The primary directional parameters are considered in many cases to include simply the mean direction, the spreading, and information on possible multi-modal peaks and on frequency dependent (or bimodal) spreading. Further details may sometimes be relevant, and plots of estimated spectra often provide helpful information. More detailed experimental and numerical investigations are needed for quantifying these general considerations for a broad range of applications.

In many cases, unidirectional wave modelling will normally be a first step, since this is easier and "cleaner" to compare directly to numerical and theoretical models. Thus there is also a challenge to develop numerical wave and structural response models taking properly into account directional effects, which will be consistent in the comparison to experiments.

2.2 The inherent statistical nature

One of the basic challenges connected with directional spectrum modelling is that, in most applications, the characteristic directional parameters are of a statistical nature, and must be interpreted as such. Thus a "unique" sample estimate is difficult to define. Estimated results are inherently subject to statistical errors on both frequency and direction (e.g. Olangon et al. 2013).

Frequency errors are related to the effect of windowing and non-stationarity. The effect of windowing can be estimated to some extent by convolution if the characteristics of the window are known. The effect of non-stationarity can be relevant in some cases. Both the mean frequency and significant wave amplitude may change during the record. For wind sea, the sweeping of the mean frequency is relatively small compared to the frequency bandwidth. For a swell it could be different and it depends on the duration of the record. For the direction, errors may arise when fitting the data with given spreading functions, which can be an ill-conditioned process.

There are basic and practical limitations in what resolution is in fact possible to document from a test. Certain characteristics of the estimates will then also be coloured by the method actually used, which is therefore important to document. This should be kept in mind, and the quantification of such errors and limitations is also a challenge.

3. MAIN DEFINITIONS

Directional frequency spectrum:

The combined frequency/directional spectrum can be written as:

$$\mathrm{S(f,\theta) = D(f,\theta)\,S(f)}$$

Here $D(f,\theta)$ is the normalized directional spreading distribution, and $S(f)$ is the scalar power spectrum including all directions. There is experimental evidence confirming the existence of bimodal directional spreading (Young et al., 1995; Ewans, 1998). However, it has been recently observed that the frequency dependent directional spread naturally develop from a initial state of no frequency dependence (Simanesew et al. 2016). Hence, for laboratory tests, it is not necessary to generate a frequency dependent directional spread since it will naturally develop, provided the wave field has been allowed to develop over a sufficiently long distance that a steady frequency dependence has been established.

Another reason why the frequency dependent directional spread is not considered in tests is because the bimodal structure usually develops beyond f/f_p =2 where the spectral energy is small compared to the peak region and thus the existence of bimodal spreading will have a little impact for engineering applications (Young et al. 1995).

If the spreading is independent of the frequency f, the spreading distribution is simplified as $D(\theta)$. The function $D(\theta)$ is often expressed as a truncated Fourier series (Longuet-Higgins et al., 1963):

$$D(\theta) = \frac{1}{\pi}\left\{\frac{1}{2} + \sum_{m=1}^{2}\left[a_m\cos(m\theta) + b_m\sin(m\theta)\right]\right\}$$

where a_m and b_m are the Fourier coefficients.

In the following, this shall be assumed unless otherwise is noted. For models of the scalar power spectrum $S(f)$ we refer to the description in ITTC (2002).

Circular moments:

$$C_m \equiv \int_0^{2\pi} D(\theta)\exp(-jm\theta)\, d\theta$$
$$\equiv a_m - jb_m \equiv |C_m|\exp(-j\varphi_m)$$

Here C_m is the complex moment of order m, and φ_m is its phase. Note that $C_0 = 1$.

Mean direction θ_0:

Two definitions are in common use:

$$\theta_{01} \equiv \arg(C_1) \equiv \arctan(b_1/a_1) \equiv \phi_1$$
$$\theta_{02} \equiv {}^1/_2 \arg(C_2) \equiv {}^1/_2 \arctan(b_2/a_2) \equiv {}^1/_2\, \phi_2$$

(notice the unit is radians). For symmetric distributions, these are identical. The second definition is less influenced by contributions outside $\pi/2 > \theta_{02} > -\pi/2$..

Spreading parameter σ_θ:

For the standard deviation, two definitions are commonly in use:

$$\sigma_{\theta 1} = [2\,(1 - \ |C_1|\,)\,]^{1/2}$$
$$\sigma_{\theta 2} = {}^1/_2\,[2\,(1 - \ |C_2|\,)\,]^{1/2}$$

The second definition is less influenced by contributions outside $\pi/2 > \theta_{02} > -\pi/2$. There are also other, model-dependent spreading parameters, defined in connection with actual models in Chapter 4.

Directional parameters (from IAHR)

A detailed list with additional directional parameters is provided in IAHR (1997) and it is copied here below for convenience:

- $\boldsymbol{k}$ [rad/s]: Wave number vector, with $|\boldsymbol{k}|=k$ being wave number, such that $k_x = \boldsymbol{k}\cos\theta$ and $k_y = \boldsymbol{k}\sin\theta$
- θ [rad]: Direction of wave propagation (describing direction of $\boldsymbol{k}$) counter-clockwise positive
- α [rad]: Wave direction, expressing where the waves are coming from. The angle is between true North and the direction where the waves are coming. Clockwise is positive in this definition
- $S_X(f,\theta)$ [m^2/Hz rad]: Directional spectral density where the subscript X is: I for incident spectrum, R for reflected, T for total or omitted is no confusion is possible
- $S_X(k,\theta)$ [(m^2/(rad/m))/rad]: Directional wave number spectral density, where X is determined as above
- $C_R(f,\theta^*)$ [ ]: Directional reflection coefficient defined as:

$$C_R(f,\theta^*) = \sqrt{\frac{S_R(f,\theta_R)}{S_I(f,\theta_I)}}$$

where $\theta_I = \pi + \theta_S - \theta^*$, and $\theta_R = \pi + \theta_S - \theta^*$, where θ^* is the deviation from the head on direction.

- $D_X(f,\theta)$ [rad^{-1}]: Directional spreading function defined as $S_X(f,\theta) = S_X(f)D_X(f,\theta)$ where

$$\int_0^{2\pi} D_X(f,\theta)d\theta = 1$$

- $R_\eta(\boldsymbol{r},\tau)$ [ ]: Autocorrelation function in space and time, i.e. the normalized autocovariance in space and time domain
- $R_\eta(\boldsymbol{r})$ [ ]: Autocorrelation function in space, i.e. the normalized in space domain
- $S_{\eta LB}(f,\theta)$ [m^2/(Hz rad)]: Directional group bound low frequency spectral density
- $S_{\eta HB}(f,\theta)$ [m^2/(Hz rad)]: Directional group bound high frequency spectral density
- $S_X(\boldsymbol{k})$ [m^2/(rad/m)^2]: Wave number vector spectral density, where X is I, R or T

- θ_s [rad]: Seaward direction of the normal to a reflecting structure
- $\theta_{m,X}$ [rad]: Mean wave direction as a function of frequency.

Def. 1: $\theta_{m,X} = \arg(c_1)$ where

$$a_n = \int_0^{2\pi} D_X(f,\theta)\cos n\theta\, d\theta$$

$$b_n = \int_0^{2\pi} D_X(f,\theta)\sin n\theta\, d\theta$$

$$c_n = a_n + ib_n$$

Def. 2: $\theta_{m,X} = \int_{\theta_{mX}-\pi}^{\theta_{mX}+\pi} D_X(f,\theta)\,\theta\, d\theta$ where X may be I or R. ***Caution***: selection of integration limits is crucial. This definition may need iteration and may converge to $\theta_{m,X}+\pi$.

In general situations, Def. 1 should be used instead of Def. 2 to avoid these potential problems.

- $\bar{\theta}_X$ [rad]: Overall mean wave direction by

  Def. 1: $\bar{\theta}_X = \arg\left(\int_{f_1}^{f_2} \frac{S_X(f)}{m_{0,X}} \exp(i\theta_{m,X})\, df\right)$
  or

  Def. 2: $\bar{\theta}_X = \int_{f_1}^{f_2} \frac{S_X(f)\theta_{m,X}}{m_{0,X}}\, df$

In general situations, Def. 1 should be used instead of Def. 2

- $\sigma_{\theta,X}(f)$ [rad]: Directional spreading (width), describing the directionality of short-crested waves

  Def. 1: $\sigma_{\theta,X}^2 = 2(1-|c_1|)$ where

$$|c_1|^2 = \left(\int_0^{2\pi} D_X(f,\theta)\sin\theta\, d\theta\right)^2 + \\ + \left(\int_0^{2\pi} D_X(f,\theta)\cos\theta\, d\theta\right)^2$$

or

Def. 2:

$$\sigma_{\theta,X}^2 = \int_{\theta_{m,X}-\pi}^{\theta_{m,X}+\pi} D_X(f,\theta)\left(\theta-\theta_{m,X}\right)^2 d\theta$$

- $\bar{\sigma}_{\theta,X}$ [rad]: Directional mean spreading, example by

$$\bar{\sigma}_{\theta,X} = \int_{f_1}^{f_2} \frac{S_X(f)\sigma_{m,X}}{m_{0,X}}\, df$$

- *UI* [ ]: Uni-directivity indes. Describing the variation of $\theta_X(f)$ with frequency. If $\theta_X(f)$ is independent of frequency, then $UI=1$. Otherwise $UI<1$

$$UI = \mathrm{mod}\left(\int_{f_1}^{f_2} \frac{S_X(f)}{m_{0,X}} \exp(i\theta_{m,X})\, df\right)$$

- $\gamma_X(f)$ [ ]: Skewness of directional spreading function, where X is I or R,

$$\gamma_X(f) = -n_{2,X}\left(\frac{1-m_{2,X}}{2}\right)^{-3/2}$$

Where

$$m_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos\left(2\left(\theta-\theta_{m,X}\right)\right) d\theta$$

$$n_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\sin\left(2\left(\theta-\theta_{m,X}\right)\right) d\theta$$

- $\delta_X(f)$ [ ]: Kurtosis of directional spreading function, where X is I or R,

$$\delta_X(f) = \frac{6 - 8m_{1,X} + 2m_{2,X}}{4(1 - m_{1,X})^2}$$

where

$$m_{1,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos(\theta - \theta_{m,X})\,d\theta$$

$$m_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos(2(\theta - \theta_{m,X}))\,d\theta$$

- W_{cw} [ ]: Largest width of 3D wave crest measured between two zero-crossings.

4. MODELLING OF MULTIDIRECTIONAL IRREGULAR WAVE SPECTRA

4.1 Input spectra and parameters

4.1.1 Uni-modal directional distribution models:

For single-peaked directional spreading, the most commonly used model is the cosine-shaped type, see below. There are also other models suggested in the literature. They are all symmetric, and in analysis of actual measured data it may be difficult to distinguish some from the others, depending on the actual spectral resolution. Thus the main characteristics often reduces to the mean and the spreading (standard deviation or similar). But the actual input model should always be documented.

Cosine model:

$$D(\theta) = A_1 \cos^{2s}[(\theta - \theta_0)/2]\,,\ \ \pi > \theta - \theta_0 > -\pi$$

where $A_1 = \{2^{2s-1}\Gamma^2(s+1) \,/\, [\pi\Gamma(2s+1)]\}$

Here the exponent s defines the spreading. A similar version often used is:

$$D(\theta) = A_2 \cos^{2N}(\theta - \theta_0)\,,\ \ \pi/2 > \theta - \theta_0 > -\pi/2$$

where $A_2 = \{\Gamma(N+1) \,/\, [\sqrt{(\pi)}\Gamma(N+½)]\}$; and N describing the spreading.

Normal (Gaussian) model:

$$D(\theta) = [1/\sqrt{2\pi}\sigma_\theta]\exp[-(\theta - \theta_0)^2/(2\sigma_\theta^2)]$$

The shape is quite similar to the cosine model (for the same standard deviation σ_θ).

Wrapped Normal:

Strictly speaking, the Normal model is defined for an infinite linear domain, and not on a circle. For narrow distributions, this is no practical problem, but a wrapped circular model is basically more correct.

Poisson model:

The Poisson model is the simplest single-peak shape of a "Maximum Entropy" model (see 4.4.2 below). It has a sharper peak but longer tails than the cosine model (for the same standard deviation σ_θ).

Other models:

Some additional models are presented and compared to the above models in an interesting review by Krogstad & Barstow (1999).

4.1.2 Multi-modal distributions:

Multi-modal distributions are normally specified as a combination of uni-modal shapes defined in 4.1.1 above.

4.1.3 Frequency-dependent spreading

Mitsuyasu model

In the Mitsuyasu model a frequency-dependent spreading exponent is assumed with s:

$$s(f) = \begin{matrix} s_p(f/f_p)^5 & \text{for } f < f_p \\ s_p(f/f_p)^{-2.5} & \text{for } f \geq f_p \end{matrix}$$

where s_p and f_p are the spreading exponent and the frequency at the spectral peak, respectively.

Donelan model

The Donelan model has the form (Donelan et al., 1985, Janssen, 2004):

$$S(f,\theta) = \frac{1}{2} S(f)\beta \cosh^{-2}[\beta(\theta - \bar{\theta}(f))]$$

where $\bar{\theta}(f)$ is the mean wave direction and

$$\beta = \begin{cases} 2.61\,(f/f_p)^{+1.3} & 0.56 < f/f_p < 0.95 \\ 2.28\,(f/f_p)^{-1.3} & 0.95 < f/f_p < 1.6 \\ 1.24 & f/f_p > 1.6 \end{cases}$$

Ewans model

Ewans (1998) proposed

$$D(f,\theta) = \frac{1}{\pi}\left\{\frac{1}{2} + \sum_{n=1}^{\infty}[a_n(f)\cos n\theta + b_n(f)\sin n\theta]\right\}$$

where:

$$\theta_1(f) = \arctan\left(\frac{b_1(f)}{a_1(f)}\right)$$

$$\sigma_1(f) = \{2[1 - (a_1^2(f) + b_1^2(f))^{1/2}]\}^{1/2}$$

are the mean wave direction and the circular rms spreading. Similarly,

$$\theta_2(f) = \frac{1}{2}\arctan\left(\frac{b_2(f)}{a_2(f)}\right)$$

$$\sigma_2(f) = \left\{\frac{1}{2}[1 + (a_2^2(f) + b_2^2(f))^{1/2}]\right\}^{1/2}$$

denote the dominant wave direction and the directional spreading factor, respectively.

Wind and swell

Sea states are sometimes defined as a combination of a wind sea component and a long-periodic swell component. They are often specified as a combination of two uni-directional spectra, collinear or in different directions. A more appropriate model is the combination of two separate directional frequency spectra $S(f,\theta) = S_{wi}(f,\theta) + S_{sw}(f,\theta)$. Here the directional distributions of each component may be frequency independent.

Generally, the two-peaks models by Ochi-Hubble or Torsethaugen are employed in tests. However, those models may be inadequated in the presence of multiple swells. In these cases, a triangular shaped spectrum is more representative for the swells (Olangon et al., 2014)

$$T(f) = \frac{2\mu(\mu-1)}{2\mu-1}\frac{H_S^2}{16 f_p}\left(\mu\frac{f}{f_p} - (\mu-1)\right)\frac{\mu-1}{\mu} f_p < f < f_p$$

$$T(f) = \frac{2\mu(\mu-1)}{2\mu-1}\frac{H_S^2}{16 f_p}\left(\mu - (\mu-1)\frac{f}{f_p}\right) \quad f_p < f < \frac{\mu-1}{\mu} f_p$$

$$T(f) = 0 \quad elsewhere$$

where

$$\mu = \frac{3Q_p + 2}{4} \qquad Q_p = \frac{2}{m_0^2}\int_0^\infty f\, S^2(f)\, df$$

4.1.4 Record duration

The accuracy and directional resolution in the final analysis increases with increasing record duration. Generally, for laboratory modelling, long records, e.g. three hours (full scale), are recommended.

4.1.5 Stationarity

It is generally assumed that stationary conditions are to be modelled. However, if modelling of time-varying conditions is wanted, it can be specified by a sequence of shorter stationary conditions.

4.2 Generation

4.2.1 Synthesizing method

The correct generation of the most realistic multidirectional irregular wave spectra in the testing area is the starting point for a meaningful experimental campaign. First of all, the inherent limitations of the basin wave field have to be clearly identified. Moreover, all the realistic aspects of the wave field have to be modelled at sufficient accuracy involving the wave maker control, flap geometries and appropriate analysis techniques. An overview of recent developments in advanced basin wave modelling including a wide range of aspects as realistic wave spreading, deterministic wave generation, focusing waves, directional wave analysis, spurious waves and shallow water wave generation is provided in Schmittner et al. (2013).

In the following the two different philosophies employed for the synthesization of multidirectional irregular wave input signals, i.e. Single-summation and Double-summation, are discussed.

Single-summation method

Control signals to the wavemaker are made to produce wave elevation time series signals $\eta(\mathbf{r},t)$ at a location $\mathbf{r}\equiv(x,y)$ according to a single summation over NF discrete frequencies f_m:

$$\eta(x,y,t) = \sum_{m=1}^{NF} A_m \cos[2\pi f_m t - k_m(x\cos\theta_m + y\sin\theta_m) + \varepsilon_m]$$

where A_m is the amplitude of the m^{th} wave component with a uniformly distributed random phase ε_m and is given by:

$$A_m = \sqrt{2S(f_m)\Delta f}$$

k_m the wavenumber, one for each frequency, with a direction θ_m randomly drawn from the actual input distribution model, and with $k_m = 2\pi/\lambda_m$, λ_m = wave length. For the Single-summation method, Yu et al. (1991) claimed that the division number of frequencies should be greater than 1000 for stable simulation.

Double-summation method

Control signals are generated according to a double summation over NF frequencies f_m and ND directions θ_n:

$$\eta(x,y,t) = \sum_{m=1}^{NF}\sum_{n=1}^{ND} A_{mn} \cos[2\pi f_m t - k_m(x\cos\theta_n + y\sin\theta_n) + \varepsilon_{mn}]$$

where A_{mn} is the component waveamplitude with a uniformly distributed random phase ε_{mn} and is given by:

$$A_{mn} = \sqrt{2S(f_m,\theta_n)\Delta f\Delta\theta}$$

Here k_m = wavenumber, one for each frequency, with a direction θ_{mn} randomly drawn from the actual input distribution model, with $k_m = 2\pi/\lambda_m$, λ_m = wavelength. The number ND of directions per frequency must be high in order to avoid non-ergodic conditions in the sea. Experience has shown that ND=100 works fine.

For the Double-summation method, Stansberg (1986) found that at least 100,000 wave components are needed for eliminating the artificial phase locking phenomenon (Jefferys, 1987) to reduce the variability of the directional spectrum to a reasonable level (see also Sand and Mynett, 1987). That is why the Single-summation method is more popular in practice than the Double-summation method for wave basins. On the other hand, the double-summation method generates a wave field with "natural" statistical variations in space. The single-summation method gives a constant, or deterministic, spectrum *S(f)* in space (if possible laboratory reflections and diffraction are disregarded).

4.2.2 White noise generation

Another approach to generate irregular sea states is to use a digital white noise *w(t)*, characterized by a density content *W(f)*. By definition of white noise, the power spectral density is $S_w(f)=1$. An example is provided in Cuong et al. (1982) which is briefly summarized below.

Given the characteristic function of the wave generator, *H(f)*, the problem is to find a function *y(t)* to be used as input to the wave maker in order to obtain the desired spectral density function to be realized, S_z. The idea behind the white noise generation approach is that the function *y(t)* can be obtained by *w(t)* through a specific filter *Q(f)*. The filter *Q(f)* may be viewed as the inverse of that needed for whitening the function *y(t)*.

Hence, if Z(f) is the desired frequency content of the wave system to be generated, it is obtained as:

$$Z(f) = H(f) \cdot Q(f) \cdot W(f)$$

and thus the frequency content of *y(t)* is

$$Y(f) = Q(f) \cdot W(f)$$

Correspondingly, the spectral density functions are related by:

$$S_z(f) = |H(f)|^2 |Q(f)|^2 S_w(f)$$

As already stated, S_w can be assumed to be unity and then:

$$S_z(f) = |H(f)|^2 |Q(f)|^2$$

which leads to:

$$|Q(f)|^2 = \frac{S_z(f)}{|H(f)|^2}$$

By introducing the additional constraint that *Q(f)* has to be a real function, the above equation finally provides

$$Q(f) = \frac{\sqrt{S_z(f)}}{|H(f)|}$$

and then:

$$Y(f) = \frac{\sqrt{S_z(f)}}{|H(f)|} W(f)$$

which represents the Fourier transform of the wave maker control time history.

The white noise approach has the advantage of generating non-repeating records.

4.3 Laboratory measurement

4.3.1 Wave elevation array

The most commonly used measuring device for estimation of directional spectra in a wave basin is an array of wave gauges for recording of the wave elevation $\eta(\mathbf{r},t)$, arranged in a particular manner. The optimal range for the interspacing between the wave gauges is more or less given by the actual wave length range - thus it is typically a fraction (≈1/10 – 1/2) of the dominant wavelengths. Too coarse arrays lead to aliasing effects, while too small arrays give very small signals relative to noise. The detailed array arrangement can vary from basin to basin –

linear array as well as circular (including triangular) configurations are frequently in use. The spatial resolution of the estimates generally increases by increased number of wave gauges, but this rule must also be seen in relation to basic limitations due to statistical variability from short records etc.

4.3.2 Particle velocities

Another common laboratory method is the combination of the horizontal particle velocity components (u_x & u_y) and elevation measurements at the same location. This resembles, in principle, measurements by pitch-and-roll buoys used in the field. The maximum possible directional resolution is less than what is in principle possible from a large array, since only the first two complex circular moments C_1, C_2 can be derived, but for short-duration records there may be only small differences.

4.3.3 Pressure array

An array of pressure sensors is sometimes used in the same manner as for elevation. In finite and shallow water, the sensors may be mounted on the bottom.

4.4 Analysis and documentation

4.4.1 Cross-spectral analysis

The common way to analyze directional spectra from a combination of irregular elevation, velocity or pressure records is by means of cross-spectra between available pairs of measuring channels. Inherently, this means that spectral averaging is an essential factor in the analysis, and statistical variability must be taken into account. From the cross-spectrum estimates, directional spectra and/or directional parameters (including some of the circular moments C_m) can then be derived by various methods, some of which are addressed in 4.4.2 below.

4.4.2 Estimation methods

Basic characteristics of some frequently used methods are briefly described in the following. More details are given in IAHR (1997), where also additional methods are described. Furthermore, new versions or updates of the methods are frequently being established. It is important to recall that the various methods are all different ways of trying to extract information from a statistical data set (cross spectra) with sampling variability, and that it may sometimes be hard to judge which is "best". In actual applications, it is important to document which method was used.

Parameters from circular moments

A simple, but robust and often satisfactory way of estimating the directional spectrum from measurements is by simple parameters such as the mean direction θ_0 and standard deviation σ_θ. As shown in Chapter 3, there are two common definition sets for these parameters, based on the estimated first and second circular moment, C_1 & C_2, respectively. Use of both sets can be helpful, since the first moment is more influenced by possible basin reflections.

In addition, information on the deviation from a symmetric cosine (or Normal) distribution shape is also of interest, in particular to check possible secondary peaks in the spectrum. Parameters that partly describe this include the skewness and kurtosis, which can also be derived from C_1 & C_2.

Distributions by parametric models

Plots of estimated directional spectra are often made by assuming a parametric model, e.g.

a cosine model or a Normal model. Parameters are estimated and then used in the plotting. The shapes will be influenced by the actual model assumed. The approach is used assuming uni-modal as well as bi-modal spectra. However, for the fine resolution of multi-peaked distributions, other methods may be preferred.

Note that the generation of directional plots directly based on a Fourier sum of the circular moments C_m is not recommended if only C_1 & C_2 are estimated, due to significant truncation effects.

Maximum Entropy Method (MEM)

Several of the most commonly used laboratory methods for estimation of distribution shapes are based upon the principle of Maximum Entropy from the theory of probability. The approach makes use of the similarity between the directional distribution function and a probability density function. Various types have been developed since the 1980's; some of them are quite advanced. They seem to reproduce multiple spectral peaks reasonably well, but as for parametric models (and all other methods as well), the estimates will be coloured by the actual method and the way it is applied. Some versions may produce too high and too many peaks in the spectra, although the most advanced versions are more reliable.

Maximum Likelihood Method (MLM)

Another widely used group of methods is based on the principle of Maximum Likelihood, originally applied in seismic detection. It is normally easier to use and more computationally efficient than MEM (van de Berg, 2011).

Bayesian method

This is based on the Bayesian technique in probability theory. It is relatively complex in numerical implementation, but at the same time it is basically a powerful method and takes consistently into account the statistical nature of the estimation problem. Any kind of single- or multi-peaked spectrum can be analysed. It is best fitted for use with multi-gauge arrays, and not so well for single-point estimation. No a priori shape or shape characteristics is inherently assumed in the Bayesian principle.

4.4.3 Multi-modal peaks; directional resolution

It should be noted that regardless of the estimation methods described above, the real resolution of multiple peaks is also limited by the statistical errors in the cross-spectral estimates, given by the record length. Thus testing the method using "ideal" cross-spectra only is helpful, but should be accompanied by tests on "real" records.

4.5 Spatial homogeneity; reflections

In laboratory modelling, there are two basic challenges in generating a spatially homogenous multidirectional sea state. First, the finite lengths of multi-flap wave-makers mean that the range of possible directions at a given point in space may depend on the actual location, at least at the outer regions of the field. There are methods in order to reduce these effects, e.g. by making use of sidewall reflections. Furthermore, reflections from beaches and sidewalls may also represent a problem, and one should try to keep these low. In any case, it will be helpful to document the magnitudes of the above effects.

5. KEY PARAMETERS

See Chapter 3 – Definitions.

6. REFERENCES

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