ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

General Guideline for Uncertainty Analysis in Resistance Tests

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General Guideline for Uncertainty Analysis in Resistance Tests

1. PURPOSE OF GUIDELINE

A general guide is provided for the practical implementation of JCGM (2008) or Guide to the Uncertainty in Measurement (GUM) and the ITTC Procedure 7.5-02-01-01 (2014a) for uncertainty analysis of measurement of resistance tests in a towing tank that follow the ITTC Procedure 7.5-02-02-01 (2017a), "Resistance Test". Analysis of the uncertainties related to extrapolation and full-scale prediction of resistance is not included in this guideline.

2. MEASURANDS

The measurands that are measured directly in resistance tests include the total resistance (R_T) and the corresponding running sinkage and trim of a ship model at each towing speed. Water temperature during model testing should be recorded to determine the density and viscosity of water.

The measured resistance is usually non-dimensionalised as the total resistance coefficient, C_T, by the following equation:

$$C_{\mathrm{T}} = 2\,R_{\mathrm{T}}/(\rho V^2 S) \qquad (1)$$

where, S is the wetted surface area of model ship, V the towing speed and ρ is the water density at the temperature during testing.

Usually, during the whole process of a typical set of resistance tests (e.g., within one day), the water temperature can be considered almost constant for a conventional indoor towing tank. If a small but significant variation occurs in temperature (much greater than 0.1 °C) with different time and area of water in testing, all the temperature measurements (t_i) should be averaged to obtain the mean temperature $\bar{t}$ as the nominal temperature for the resistance tests. Each resistance measurement at temperature (t_i) should first be converted to the average temperature $\bar{t}$ before any data analysis is performed.

When some tests are repeated or intra/inter-laboratory comparison is performed, if a deviation in towing speed occurs, the resistance measurement should also be corrected to the nominal speed that corresponds to the prescribed Froude number

$$Fr = V/\sqrt{gL} \qquad (2)$$

where L is the characteristic length of the model and g is the local acceleration of gravity. The uncertainties of the conversion and correction mentioned above are not included in this guideline.

Additionally, the blockage effect of the tank boundaries can be corrected with use of one of the formulae recommended in ITTC Procedure 7.5-02-02-01 (2017a). These formulae are all based on mean-flow theory under some assumption and with some simplicity. Uncertainty of such correction is not included in this guideline. Typically, the blockage effect of a large deep towing tank is negligible, and usually no correction is needed.

3. UNCERTAINTY SOURCES

The first step in implementing the GUM into the resistance tests is to identify all the significant sources of uncertainty, mainly on the basis of the database or practical judgment by well-experienced engineers in towing tank.

Along the whole flowchart of resistance test, the sources of uncertainty in measurement may be grouped under five blocks from No. ① to No. ⑤ as shown in Figure 1. Each group of uncertainty sources is outlined in the following sections.

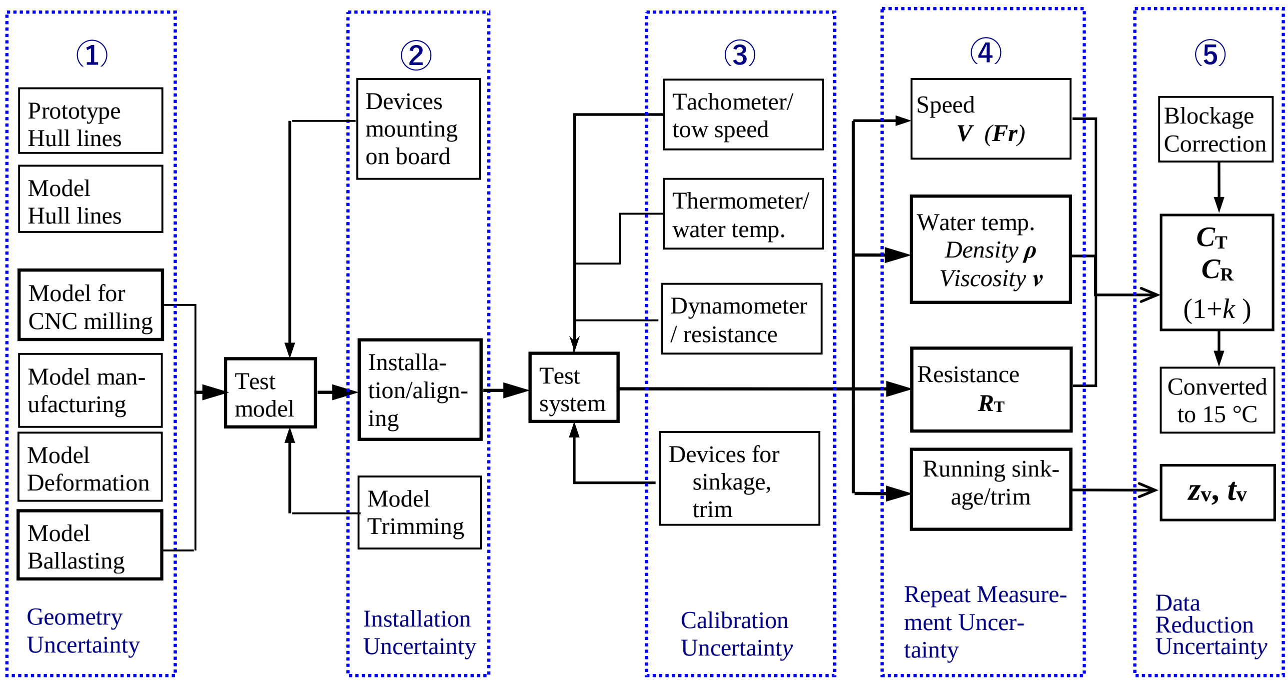

Figure 1: Schematic diagram for groups of uncertainty sources in resistance test

3.1 Model Geometry

No. ① block in Figure 1 lists the uncertainty sources related to model geometry. The geometry uncertainty mainly results from the tolerances in manufacturing and the deformation after manufacture and during model testing.

The wetted surface area is an important parameter in resistance data reduction. To determine accurately the real wetted surface of a hull model is difficult under testing because of the effect of hull-making waves and running attitudes. Instead, a nominal area, i.e., the wetted surface area of a hull model at rest, is usually adopted and theoretically computed from the hull form lines.

Usually, a hull model is manufactured by a multi-axis Computerized Numerical Control (CNC) milling machine. The nominal wetted area can be numerically computed by surface integral of the 3D numerical model for CNC milling. However, a slight difference in the surface fairing of hull model will occur between different workshops, when the same hull form lines are used for manufacture. This difference is not an uncertainty in the hull geometry, but rather it is a definite bias between workshops. However, laser technology is available for the measurement of the hull relative to the CAD drawings, which can provide an estimate in the uncertainty in the model dimensions and the wetted surface area.

The hull model is to be ballasted in accordance with its nominal displacement volume, which can be obtained by integrating the surface of 3D numerical model up to the nominal waterline/draught, that is,

$$\Delta = \rho\nabla \tag{3}$$

where ρ is the water density and ∇ is the displacement volume.

Therefore, the uncertainty in model ballasting will propagate into the real displacement volume of hull model. The relative expanded uncertainty can be expressed as

$$U_\Delta/\Delta = U_\nabla/\nabla \tag{4}$$

where U is the expanded uncertainty, $U = ku$ and k is the coverage factor, and u the standard uncertainty.

The displacement volume of a hull model represents a sort of "size" of the wetted part of hull model. The representative length (L) and area (S) for non-dimensional coefficients can be assumed proportional to one-third and two-third power of the volume, respectively,

$$\begin{aligned} L &\propto \nabla^{1/3} \\ S &\propto \nabla^{2/3} \end{aligned} \tag{5}$$

Therefore, the uncertainty components of wetted surface area and representative length of the model can be estimated as,

$$\begin{aligned} U_L/L &= (1/3)\,U_\nabla/\nabla \;= (1/3)\,U_\Delta/\Delta \\ U_S/S &= (2/3)\,U_\nabla/\nabla \;= (2/3)\,U_\Delta/\Delta \end{aligned} \tag{6}$$

where, the length uncertainty will propagate into frictional resistance calculation through the Reynolds number,

$$Re = VL/\nu \tag{7}$$

where ν is the kinematic viscosity of water.

The uncertainty in trimming the hull model is assumed to have no effect on the wetted surface area, S, of the model; however, the difference can be estimated by calculation of S at the upper and lower limit of the waterline. The thermal deformation of the hull model due to the change of ambient temperature between the model workshop and tank water is usually assumed negligible, as the coefficient of thermal expansion (CTE) of wood is small, on the order of 5 x 10^{-5}/°C, and variation of temperature can keep within several degrees at indoor laboratory from model manufacturing to model testing. No analytic relationship exists between the non-uniform deformation of hull geometry and the hull resistance (especially the form drag), let alone the effect of the waviness of hull surface on the resistance. Model expansion may be affected by moisture content depending on the model material; however, any dimensional changes can be measured with laser measurement technology.

Finally, from Equation (3), the uncertainty of water density propagates into the real displacement volume of hull model in tank water, i.e.

$$U_\nabla/\nabla = U_\rho/\rho = (\partial\rho/\partial t)\,U_t/t \tag{8}$$

where t is the temperature and the sensitivity coefficient, $c_\rho = \partial\rho/\partial t$, is tabulated in ITTC Procedure 7.5-02-01-03 (2011).

Usually, the temperature of tank water during a set of routine tests varies very little, say, much less than ± 0.5 °C. From ITTC Procedure 7.5-02-01-03 (2011), the sensitivity coefficient is 0.151 kg/m^3·°C at 15 °C and water density of 999.1026 kg/m^3 or a change of ± 0.0076 % in the water density. The uncertainty component result in geometry is considered negligible. However, when the same model is tested at different dates and the water temperature changes several degrees, the model hull should be ballasted to the water temperature corresponding to the specific date.

Although the contribution to the uncertainty in displacement from water density can be insignificant, the uncertainty in the location of the waterline can be. The uncertainty in the volumetric displacement is simply the uncertainty in the location of the waterline times the waterplane area

$$U_\nabla = A_\mathrm{W} U_T \tag{9}$$

$$U_\Delta = \rho A_\mathrm{W} U_T \tag{10}$$

where A_W is the area at the waterplane and U_T is the uncertainty in the draught or waterline location. For a model or full-scale ship, the uncertainty can be on the order of ±1 %. The area at the waterplane can be computed from the CAD drawing.

Model dimensions may also be measured with laser technology. The standard deviation of model dimensions. s_L, are reported relative to the CAD drawings of the model. The expanded uncertainty of the model length, L, is

$$U_L = 2 \cdot s_L \tag{11}$$

The wetted surface area is proportional to the length squared, L^2. The expanded relative uncertainty in wetted surface area from laser measurements is then

$$U_S/S = 2U_L/L = 4 \cdot s_L/L \tag{12}$$

3.2 Test Installation

No.② block in Figure 1 includes the uncertainty sources related to the hull model trimming, the alignment of the centreline of the hull model and dynamometer and the motion direction of the towing carriage, the alignment of tow force to the line of propeller shaft, and so forth.

Usually, the installation process in commercial tanks can be controlled so well that the uncertainty from installation into the hull model resistance is assumed to be negligible, when no reliable database is available to estimate such uncertainty in resistance measurements.

No direct method is available to evaluate analytically the effect of non-zero drift angle of the hull model resulting from the uncertainty of alignment on the hull resistance. Consequently, this effect is usually assumed negligible. With dynamometer systems that measure side-force for symmetrical models, a series of runs at different angles of drift can be run to assess possible misalignment. Overall, this is an area where good practice is essential.

3.3 Instrument Calibration

The devices for measuring the tow speed of carriage, the running sinkage and trim, and the temperature of tank water are all calibrated regularly. The uncertainties given in the calibration reports or certificates can be directly quoted for the resistance tests. However, those uncertainty values should be firstly converted into the corresponding expanded uncertainties according to the GUM.

The resistance dynamometer is usually calibrated before test and checked immediately after a test. All the calibration results should be traceable to a National Metrology Institute (NMI).

The dynamometer calibration should be performed according to the ITTC Procedure 7.5-01-03-01 (2017b). At least, ten equal increments of loads or forces over the range are adopted for end-to-end or through system calibration process, in which the signal conditioner and data acquisition system (DAS) are all included. Analysis of the details of uncertainty components from inside DAS is not necessary, unless some improvement in the measuring system is made.

Additionally, ITTC Procedure 7.6-02-09, Revision 01 (2020), recommends calibration with two measurements over 10 increments. The procedure also proposes random loading. Usually, loading is sequential with adding weights to the maximum load followed by removing weights from the maximum load to zero. The dynamometer should be loaded to the maximum load twice before data are recorded.

Linear curve fitting is always applied for dynamometer calibration. The method in ITTC Procedure 7.5-01-03-01 (2017b) should be applied with an uncertainty estimate at the 95 % prediction limit from calibration theory. Typically, the uncertainty is about 3·*SEE*, where *SEE* is the standard error of estimate from linear regression analysis. On a calibration stand, the dynamometer is calibrated by changing precision weights. As a minimum the weights should be OIML (2004) class M_2. The following is the data processing equation for conversion from mass to force

$$F = mg(1 - \rho_A/\rho_M) \qquad (13)$$

where m is the applied mass, g local gravity, ρ_A air density, and ρ_M mass density of the weight. The last term is a buoyancy correction. The following are the values for Equation (13)

g 9.8066 m/s^2 for standard gravity
ρ_A 1.2 kg/m^3
ρ_M 8000 kg/m^3

The tolerance for the weight set should be applied as the uncertainty estimate to the total mass, m. The uncertainty in force from the weights is negligible. Local gravity is typically less than standard gravity. The corrections for local gravity and buoyancy are each typically larger than the weight tolerance.

The uncertainty of the calibration will consist of three elements:

- Uncertainty in the applied force for each data point
- Standard deviation of the time series from the DAS for the Type A evaluation for each data point
- Uncertainty in the curve-fit from the 95 % prediction limit

For the calibration of a dynamometer, the first two items are negligible, and most of the uncertainty is in the curve-fit.

Sinkage and trim are typically measured by a pair of string potentiometers located forward and aft locations on the model. However, sinkage may be measured with a single string potentiometer located at the CG with a precision electronic inclinometer for trim. By either method the devices should be calibrated in a manner similar to the dynamometer calibration by ITTC Procedure 7.5-01-03-01 (2017b).

3.3.1 Outliers

Calibration data should be evaluated for outliers and systematic deviations from linearity. A simple check is to plot the data as a residual plot, that is, the difference between the curve fit and the data. The residuals should be plotted as the standardized residuals, the residuals divided by the *SEE*. Acceptable data will be randomly scattered about zero. Any data larger than 2·*SEE* may be a suspect as an outlier.

Outliers are identified by Chauvenet's criterion in Figure 2 from Coleman and Steele (2009). As an example for 10 data points, the threshold is 1.96. Figure 2 may also be applied to a time series or repeat measurements where the abscissa is $(\bar{x} - x_i)/s_x$.

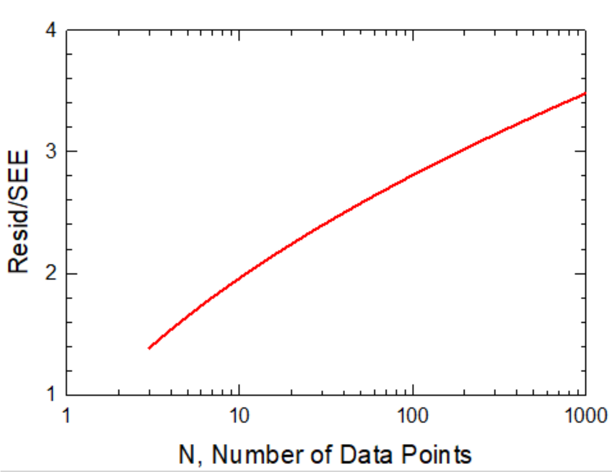

Figure 2: Chauvenet's rejection criterion for outlier data

3.4 Direct Measurement

3.4.1 Resistance

No.④ block in Figure 1 indicates the uncertainty sources related to the measuring data that are directly output from DAS of measurement system. The effect of DAS on uncertainty of resistance measurement is preferably included in the calibration by end-to-end calibration, but some special consideration should be given in the time history of sampling data.

Usually, the time history in an interval of time, $\Delta t = n/f_s$, is obtained after low-pass filtering, where f_s is the sampling rate and n the number of the sampling data points. The sampling rate should be at least double the low-pass filter setting. Thereafter, the filtered time history is averaged to obtain one "reading" of the measurement,

$$\hat{R}_{\mathrm{T}} = (1/n)\sum_{i=1}^{n} R_{\mathrm{T}i} \quad (14)$$

and the standard deviation of the filtered time history is calculated as

$$\hat{s}_{R_{\mathrm{T}}}^2 = [1/(n-1)]\sum_{i=1}^{n}(\hat{R}_{\mathrm{T}} - R_{\mathrm{T}i})^2 \quad (15)$$

where $R_{\mathrm{T}i}$ is the i-th data point within the filtered time history. Then, the expanded uncertainty of the "reading" (average) can be obtained,

$$U_{\hat{R}_{\mathrm{T}}} = k\,\hat{s}_{R_{\mathrm{T}}}/\sqrt{n} \quad (16)$$

Usually, the coverage factor is $k = 2$; however, for small sample sizes the Student-t value may be applied. The sampling rate and interval of time and the cut-off frequency of low filtering should be properly chosen so that the standard uncertainty of the average will be negligible.

Furthermore, if repeat tests are performed, the mean of multiple runs is adopted from repeat tests rather than the average of a single run as a better estimate of measurand

$$\bar{R}_{\mathrm{T}} = (1/N)\sum_{j=1}^{N} \hat{R}_{\mathrm{T}j} \quad (17)$$

where N is the number of repeat tests. The experimental standard deviation of these N runs can be estimated by the following Equation (18)

$$s_{R_{\mathrm{T}}}^2 = [1/(N-1)]\sum_{j=1}^{N}(\bar{R}_{\mathrm{T}} - \hat{R}_{\mathrm{T}j})^2 \quad (18)$$

From Equation (16), the expanded uncertainty becomes

$$U_{\bar{R}_{\mathrm{T}}} = k\,s_{R_{\mathrm{T}}}/\sqrt{N} \quad (19)$$

Since the number of repeats is likely small, the Student-t should be applied as the coverage factor, $k = t_{95}$.

Equation (19) is the 95 % confidence limit of the mean value for repeat runs. However, the 95 % prediction limit from a single run is given by

$$U = ks\sqrt{1 + 1/N} \quad (20)$$

For a large number of samples, the 95 % prediction limit is $U = 2 \cdot s$.

3.4.2 Running Sinkage and Trim

The uncertainty in direct measurement of running sinkage and trim can be analysed similarly as the above. Usually, sinkage and trim are measured with a pair of string potentiometers, one forward and the second aft. The average running sinkage is given by the following from ITTC (2005):

$$z_{VM} = (z_{VF} + z_{VA})/2 \quad (21)$$

where z_{VF} is running sinkage at the forward point (FP) and z_{VA} the running sinkage at the aft point (AP) from string potentiometers.

Running trim in pitch in radians is defined from the ITTC (2005) by

$$\theta_D = \tan^{-1}(z_{VF} - z_{VA})\,/L \quad (22)$$

For small pitch angles, Equation (22) is approximately

$$\theta_D \approx (z_{VF} - z_{VA})/L \quad (23)$$

where L is the distance between the strings of the string potentiometers.

The analysis is simplified, when heave is measured with a single string potentiometer at the CG and pitch is measured with an on-board precision electronic inclinometer. Sinkage and trim are then measured directly. The uncertainty in sinkage is from a single calibration of the string potentiometer, and the uncertainty in trim is from calibration of the electronic inclinometer.

3.4.3 Water Temperature

The density and viscosity of water in towing tank are determined by water temperature and calculated according to the ITTC Procedure 7.5-02-01-03 (2011). The water temperature is usually measured with an accuracy of ±0.10 °C. The uncertainties in density and kinematic viscosity are computed as follows from the uncertainty in temperature, *t*.

$$U_\rho = (\partial\rho/\partial t)U_t = c_\rho U_t \quad (24)$$

$$U_\nu = (\partial\nu/\partial t)U_t = c_\nu U_t \quad (25)$$

where the values for the density, viscosity, and their sensitivity coefficients are obtained from the tables in ITTC 7.5-02-01-03 (2011).

The water density at 15 °C will be 999.103 ±0.015 kg/m^3 or ±0.0015 % from a sensitivity coefficient of 0.151 kg/m^3·°C and uncertainty in temperature of 0.10 °C. The sensitivity coefficient for kinematic viscosity is 3.00 x 10^{-8} m^2/s·°C. The kinematic viscosity at 15 °C is (1.1386 ±0.0030) x 10^{-6} m^2/s or ±0.26 %. The deviation of water temperature has a relatively large effect on water viscosity and thereafter on the Reynolds number and frictional drag of the hull model.

3.4.4 Carriage Speed

For a carriage test, the speed is computed from the rotation of a precisely measured diameter of a metal wheel, where the speed is calculated from

$$V = \pi DN \quad (26)$$

where D is the wheel diameter in metres (m) and N is the rotational rate in Hz. For a wheel with a digital encoder, the rotational rate is

$$N = n/(pt) \quad (27)$$

where n is the number of pulses during the carriage run, p the number of pulses per revolution for the digital encoder, and t is the run time in seconds (s).

3.5 Data Reduction

In addition to the data reduction equations for Froude number, *Fr*, and Reynolds number, *Re*, in Equations (2) and (7), respectively, and the total resistance coefficient, C_{T}, in Equation (1), the following are the equations for friction coefficient, C_{F}, and residuary coefficient, C_{R}:

$$C_{\mathrm{F}} = 0.75/(\log_{10} Re - 2)^2 \tag{28}$$

$$C_{\mathrm{R}} = C_{\mathrm{T}} - (1 + k_{\mathrm{F}})C_{\mathrm{F}} \tag{29}$$

where k_{F} is the form factor.

4. UNCERTAINTY PROPAGATION

4.1 Froude Number

From Equation (2), the sensitivity coefficients for Froude number are

$$c_V = \partial Fr/\partial V = 1/\sqrt{gL} \tag{30a}$$

$$c_g = \partial Fr/\partial g = -V/(2\sqrt{g^3 L}) \tag{30b}$$

$$c_L = \partial Fr/\partial L = -V/(2\sqrt{gL^3}) \tag{30c}$$

The uncertainty in Froude number is

$$U_{Fr} = \sqrt{(c_V U_V)^2 + (c_g U_g)^2 + (c_L U_L)^2} \tag{31}$$

Typically, the contribution from the velocity is the largest, and the contribution from g is negligible.

From Equation (26), the uncertainty in velocity from a carriage wheel is

$$U_V = \pi\sqrt{(N U_D)^2 + (D U_N)^2} \tag{32}$$

The uncertainty in length may be established with laser measurements of the model. The uncertainty should be within the model requirements by ITTC Procedure 7.5-01-01-01 (2017c) of ±0.05 % L_{PP} (length between perpendiculars) or ±1.0 mm, whichever is the largest. The uncertainty in velocity as two elements: calibration of carriage speed from Equation (32) and Type A evaluation from the standard deviation of the speed from a time series.

4.2 Reynolds Number

From Equation (7), the sensitivity coefficients for Reynolds number are

$$c_V = \partial Re/\partial V = L\,/\,\nu \tag{33a}$$

$$c_L = \partial Re/\partial L = V/\nu \tag{33b}$$

$$c_\nu = \partial Re/\partial\,\nu = -VL/\nu^2 \tag{33c}$$

The uncertainty in Reynolds number is

$$U_{Re} = \sqrt{(c_V U_V)^2 + (c_L U_L)^2 + (c_\nu U_\nu)^2} \tag{34}$$

The uncertainty in velocity and length is the same as described for Froude number. The largest contributor is likely the uncertainty in viscosity with a nominal relative uncertainty of ±0.26 %, which is computed from the uncertainty in temperature by Equation (25).

4.3 Total Resistance Coefficient

The total resistance coefficient is computed from Equation (1). The sensitivity coefficients from the equation are

$$c_R = \partial C_{\mathrm{T}}/\partial R_{\mathrm{T}} = 2/(\rho V^2 S) \tag{35a}$$

$$c_\rho = \partial C_{\mathrm{T}}/\partial \rho = -2R_{\mathrm{T}}/(\rho^2 V^2 S) \tag{35b}$$

$$c_V = \partial C_{\mathrm{T}}/\partial V = -4R_{\mathrm{T}}/(\rho V^3 S) \tag{35c}$$

$$c_S = \partial C_{\mathrm{T}}/\partial S = -2R_{\mathrm{T}}/(\rho V^2 S^2) \tag{35d}$$

where the uncertainty in density is computed from the uncertainty in temperature by Equation (24).

As indicated previously, the contribution from density will be negligible. The uncertainty in wetted surface may be relatively large. The uncertainties in velocity and resistance consist of two elements: calibration uncertainty and uncertainty by the Type A qualification from the standard deviation of the time series in the test.

4.4 Friction Coefficient

The sensitivity coefficient from Equation (28) is

$$c_{Re} = -0.87 C_F / (log_{10} Re - 2) \tag{36}$$

$$U_{C_\mathrm{F}} = c_{Re} U_{Re} \tag{37}$$

where the uncertainty in Reynolds number is from Equation (34).

4.5 Sinkage and Trim

For the sinkage and trim measurements from a pair of string potentiometers in Equation (21), the calibration of the string potentiometers is from the same equipment; consequently, the data are correlated. The resulting uncertainty from calibration is as follows:

$$U_{zV\mathrm{M}} = (U_{zV\mathrm{F}} + U_{zV\mathrm{A}})/2 = U_z \tag{38}$$

Likewise, the result in the trim angle from Equation (23) for the string potentiometer calibrations is then

$$U_{\theta D} = (U_{zV\mathrm{F}} - U_{zV\mathrm{A}})/L = 0 \tag{39}$$

Consequently, the sole contributor to the uncertainty in calibration is the length between the string potentiometers

$$U_{\theta D} / \theta_D = U_L / L \tag{40}$$

During a test, the measurements are independent for each string potentiometer. The uncertainty from the test measurements are then combined with Equations (38) and (40) in the usual manner.

The case for the single heave measurement for sinkage and a single electronic pitch measurement is much simpler. A single calibration and test measurement are provided for each sinkage and trim.

4.6 Repeat Tests

As a better estimate of the uncertainty, a typical test condition should be repeated at a minimum of ten (10) times. For the average result, the expanded uncertainty component from repeat tests can be estimated by Equation (19), where the appropriate parameter is substituted for R_T. If any single test is adopted as the final measurement, the expanded uncertainty should be estimated by Equation (20) with the standard deviation from the repeated test.

5. UNCERTAINTY SUMMARY

This procedure outlines the instrumentation, data processing equations, and related uncertainty. The sensitivity coefficients are provided for the uncertainty analysis. The following procedures should be reviewed for additional details on instrument calibration:

- ITTC Procedure 7.5-01-03-01, “Uncertainty Analysis, Instrument Calibration”
- ITTC Procedure 7.6-02-09, Calibration of Load Cells”

Procedures are available which apply this procedure to example calculations with uncertainty estimates. These include the following:

- ITTC Procedure 7.5-02-02-02.1 "Example for Uncertainty Analysis of Resistance Tests in Towing Tanks"
- ITTC Procedure 7.5-02-02-07 "Practical Implementation of Uncertainty Analysis"

6. LIST OF SYMBOLS

6.1 English

A_W	Waterplane area	m^2
C_F	Frictional resistance coefficient, Equation (28)	1
C_R	Residuary resistance coefficient, Equation (29)	1
C_T	Total resistance coefficient, Equation (1)	1
c_i	Sensitivity coefficient, $c_i = \partial f/\partial x_i$	
D	Diameter	m
F	Force	N
Fr	Froude number, Equation (2)	1
g	Local acceleration of gravity	m/s^2
k	Coverage factor, usually $k = 2$	1
k_F	Form factor	1
L	Length	m
L_{PP}	Length between perpendiculars	m
m	Mass	kg
N	Rotational rate	rad/s
N	Number of repeat runs	
n	Number of samples	1
R_T	Total resistance	N
Re	Reynolds number, Equation (7)	1
S	Wetted surface area	m^2
s	Standard deviation	
t	Temperature	°C
U	Expanded uncertainty, $U = ku$	
u	Standard uncertainty	
V	Velocity	m/s
z_{VA}	Aft running sinkage	m
z_{VF}	Forward running sinkage	m
z_{VM}	Average running sinkage, Equation (21)	m

6.2 Greek

Δ	Displacement	kg
θ_D	Running trim, Equations (22) (23)	rad
μ	Absolute viscosity	Pa·s
υ	Kinematic viscosity, $\upsilon = \mu/\rho$	m^2/s
ρ	Density	kg/m^3

6.3 Other

∇	Volumetric displacement, $\nabla = \Delta/\rho$	m^3

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