ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

Guideline to Practical Implementation of Uncertainty Analysis

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Guideline to Practical Implementation of Uncertainty Analysis

1. PURPOSE OF PROCEDURE

The purpose of this procedure is providing a beginner's introduction to the subject of experimental uncertainty and its application to the maritime engineering hydrodynamic testing industry. First, the document provides an overview of the key steps to understand uncertainty in an experimental process. The document further outlines the key subject specific knowledge areas needed for understanding. This includes identification of the key resources in the form of published material available to a beginner in the subject. The last section provides a worked example for a basic resistance test, which is intended as a learning tool for those studying the subject.

1.1 Overview of the Experimental Uncertainty Analysis Process

Ref. to para 5., ITTC 7.5-02-01-01

JCGM (2008a) classifies uncertainties into three categories: Standard Uncertainty, Combined Uncertainty, and Expanded Uncertainty.
The standard uncertainty of the result of a measurement consists of several components, which can be grouped into two types.

Type A: Uncertainty components obtained from a method based on statistical analysis of a series of observations.
Type B: Uncertainty component obtained by other means (other than statistical analysis). Prior experience and professional judgements are part of type B uncertainties.

Outlining the various stages is a first necessary step that can be undertaken in an uncertainty analysis. As a starting point, the four fundamental stages of uncertainty analysis are introduced:

- Law of propagation of uncertainty
- Repeatability test
- Reproducibility test
- Inter-laboratory comparison test.

Before elaboration of the four described stages, the following sections outline some background information.

1.1.1 Error or Uncertainty

What is uncertainty? Uncertainty is not error. The difference is described in the following.

An error, for example, may be made when conducting an experiment, resulting in an incorrect result. Such mistakes may always be possible. However, appropriate and thorough preparations and conduct of an experiment together with robust quality systems reduce the risks to acceptable levels. Hence, qualified and, most importantly, experienced personnel, a robust and comprehensive procedure, internal company procedures, and the appropriate level of oversight are required as pre-requisites.

JCGM (2008) defines the uncertainty of measurement as the "parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand." Uncertainty describes the level of precision of a measurement. If the length of a ship model has to be known within a millimetre (mm), a tape measure in millimetres can give a satisfactory result. If the measurement is needed within a fraction of a millimetre, a measuring device with a greater degree of precision is required. Additional information such as the temperature at which the measurement was made maybe needed. Whatever instrument or method is employed, only an estimate of the measurement to within some defined level of precision will be available. All measurements should be traceable to a National Metrology Institute (NMI).

For any experiment, the precision with which the various contributing measurements are taken and how these values propagate through the data reduction equations, dictate the precision in the final result.

1.2 Overview of the Stages of Analysis

The following sections provide a brief overview of the four key stages of analysis outlined in the previous sections.

1.3 Law of Propagation of Uncertainty

In the most general sense, the uncertainty in an estimated or scientifically calculated value which is the root-sum-square of the uncertainty in each input variable multiplied by the sensitivity of the result to a change in that variable.

1.3.1 Uncertainty of a Variable

Consider first the uncertainty in the variable. This can typically take one of two forms:

1. If the measured variable is a function of time, then a sufficiently large sample size is needed for statistical analysis.

For example, a ship model is towed along a towing tank, and the resistance or drag force is measured. The measured force will typically fluctuate about some mean value this measurement is applied in subsequent calculations. The uncertainty in this case could be taken as the standard deviation of the mean value and applied by the Type A evaluation method.

2. If a single measurement is taken (such as during a calibration), alternative methods have to be employed.

This may include information by the Type B method from calibration certificates, past experience or simply based on sound engineering judgment. As an example, the uncertainty in the weight of the ballasted ship model can be taken as the value given on the NMI traceable calibration certificate for the scales weighing it. That is, the calibration certificate will state that the scale is accurate to plus or minus some value which is the uncertainty.

1.3.2 Sensitivity of a Variable

The sensitivity of the result to a change in each variable is considered next. Though more sophisticated methods exist, this can actually be readily achieved by simply assigning varying realistic values to the equations and computing what happens when small changes are made to the values in question by a central finite differencing method as described in ITTC (2014a) and JCGM (2008).

If for example, a spread-sheet containing the calculations required for the analysis of the result is available, then the input variables will include a range of parameters such as (in the case of a resistance experiment): the masses and moments of masses (radius of gyration), dimensions, water properties, etcetera.

The output would be the derivation (the experimental estimate) of the ship model resistance. Following subsequent quantification of scaling issues, the full-scale ship resistance is estimated.

For each of the input variables in turn, a small change in that input variable can be made to investigate the effect in the overall result. The sensitivity coefficient is given by dividing the change in the result by the change in the input variable. This coefficient shows how sensitive the result is to a small change in the input variable in question.

1.3.3 Combined Uncertainty

After the uncertainty for each input variable has been estimated and the associated sensitivity calculated, the estimate of the combined uncertainty is possible. The first step is simply to take the squared value for each individual uncertainty and multiply it by the squared value for the corresponding sensitivity coefficient. The second step is to sum them and take the square-root of the result. In its simplest form, this is combined uncertainty.

The law of propagation of uncertainty described above is invaluable as a pre-test tool to estimate the likely uncertainty and to identify critical parameters. This can better inform the experimental design and help target investment in areas where the most benefit will be in increasing the confidence in the estimated results. Usually, the largest contribution is from one measurement. In that case, the test engineer can focus on that variable and determine if the uncertainty can be reasonably reduced.

2. REPEATABILITY TEST

By repeating the test a sufficient number of times, a better estimate of the uncertainty will be obtained. This only accounts for the uncertainty in the load and speed measurements in a resistance test. Other uncertainties, such as model parameters or water properties, still need to be included.

Repeating a test condition multiple times can be time consuming and expensive. Repeat tests are not necessary for every test and/or every client, but they should be completed for a single representative test condition. The repeatability test is a tool to help determine that the experimental test is stable and well founded. Once performed, a reasonable inference can be made for other experiment of the same type.

3. REPRODUCABILITY TEST

The repeatability test described above establishes if the process is well founded, but it does not mean that the procedures are sufficiently robust. For example, the method of the calibration of a component of the test is performed by different facilities.

To understand such influences, laboratories perform reproducibility tests. Rather than just repeating the test, the facility will repeat the whole experiment. That is, once the test is completed, the team will disassemble the whole set-up. On a later occasion, the team (or a different team) will start from scratch, set the test up, and repeat the measurements. After this has been done a sufficient number of times, a mean value for the test result can be extracted together with the corresponding uncertainty.

This process is even more expensive and time consuming than the repeatability tests. Reproducibility demonstrates not only that the experimental test is stable and well-founded but that the procedure is robust. The definitions of repeatability and reproducibility are located in the GUM, JCGM (2008).

4. INTER-LABORATORY COMPARISON TESTS

An inter-laboratory comparison test is another example of reproducibility test. Something about a facility may have an impact on the test results. By conducting the same experiment at multiple institutions, such effects can be better understood, and problems eliminated.

A well conducted inter-laboratory comparison test can provide invaluable information about the stability of an experimental process and the robustness of the procedures. An ITTC inter-laboratory resistance test for a surface ship model is described in ITTC (2014b). The details of an inter-laboratory comparison methodology are discussed in ITTC (2014a)

5. UNCERTAINTY EXAMPLE FOR A RESISTANCE TEST

This section includes worked examples to help those new to the subject become familiar with the basic steps.

5.1 Froude Number Example

The first worked example is the simple case of finding the uncertainty in a Froude number estimate. The aim of the example is to introduce those new to the subject to the fundamental process of combining the uncertainties associated with input variables and associated sensitivities and to obtain a combined uncertainty. The objective is to show an example of a practical approach for estimating variable uncertainties and obtaining the associated sensitivity terms.

The Froude number defined in Equation (1) is a function of the velocity V (scaled ship operational design speed), the acceleration of local gravity g, and a representative length such as the length between perpendiculars of the ship model given by L

$$Fr = V/\sqrt{gL} \tag{1}$$

The law of propagation of uncertainty for uncorrelated and independent measurements is given by the following for the combined standard uncertainty.

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) \tag{2}$$

where u is the standard uncertainty and the sensitivity coefficients are given by

$$c_i = \partial y/\partial x_i \tag{3}$$

The elements of Equation (2) may also be computed from a finite central differencing method from ITTC (2014a) and JCGM (2008) as

$$u_i(y) = c_i u(x_i) = [1/2][f(x_1, x_2, \cdots x_i + u(x_i), \cdots x_N) \quad - f(x_1, x_2, \cdots x_i - u(x_i), \cdots x_N)] \tag{4}$$

Equation (4) may be applied for more complicated data processing equations without computing the sensitivity coefficients from Equation (3). The method may also be useful for the checking the results from Equation (2).

The standard uncertainty by the Type A evaluation is given by

$$u = s/\sqrt{n} \tag{5}$$

where s is the standard deviation and n the number of samples or observations. The standard uncertainty is also defined as the standard deviation of the average value.

That is, Equation (2) gives the combined uncertainty where $u_c(y)$ is the combined standard uncertainty in the result, $u(x_i)$ is the standard uncertainty of the ith variable, and c_i is the sensitivity coefficient, which is the partial derivative of the result with respect to any given input variable x_i given in Equation (3).

The combined standard uncertainty $u_c(y)$ is universally applied in the expression of the uncertainty of a measurement result. Expanded uncertainty, U, from the combined uncertainty $u_c(y)$ multiplied by a coverage factor, k, is:

$$U = ku_c(y) \tag{6}$$

The result of a measurement should be expressed as $Y = y \pm U$, or the best estimate of the value attributable to the measurand Y is between (y - U and y + U). The interval $y \pm U$ may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to Y. The value of U should be provided to two significant digits with y to the same resolution. Usually, the 95 % confidence limit has a coverage factor, $k = 2$. The coverage factor may also be determined from the inverse Student-t or $k = t_{0.025,\, n\text{-}1}$.

First, the uncertainty for each variable is estimated, and the second task is determining the sensitivity coefficients. For a carriage test, the speed is computed from the rotation of a precisely measured diameter of a metal wheel, where the speed is calculated from

$$V = \pi DN \tag{7}$$

where D is the wheel diameter in meters (m) and N is the rotational rate in Hz. For a wheel with a digital encoder, the rotational rate is

$$N = n/(pt) \tag{8}$$

where n is the number of pulses during the carriage run, p the number of pulses per revolution for the digital encoder, and t is the run time in seconds (s). For the purpose of this analysis, details of the uncertainty estimate of Equations (7) and (8)will not be provided. For the example, the uncertainty of the velocity will be estimated as 0.10 % of the velocity from ITTC (2014b). Additionally, the standard deviation of the velocity from the time history for the run can be included by the Type A method.

Local g of the test facility can be computed from Moose (1986) from the latitude and longitude in the United States. PTB (2019) had published a version for the computation of local g anywhere in the world, but it was discontinued recently. An uncertainty estimate is provided by the calculation. ASTM (2018) requires that the uncertainty for force calibration be ±0.00010 m/s^2. The uncertainty from Moose (1986) is typically less than the ASTM requirement. An uncertainty in model length is applied as ±0.050 % of length from the tolerance requirement of ITTC (2017a) for surface ship models. The uncertainty can be less from measurements of the model dimensions with laser technology.

As an example, Froude number and its uncertainty estimate are computed from the model test of Longo and Stern (1998 and 2005) for Fr = 0.28. The results are summarized in Table 1.

The sensitivity coefficients for Table 1 are computed from Equations (1) and (3) as follows:

$$c_V = \partial Fr/\partial V = 1/\sqrt{gL} \tag{9a}$$

$$c_g = \partial Fr/\partial g = -V/(2\sqrt{g^3L}) \quad \text{(9b}$$

$$c_L = \partial Fr/\partial L = -V/(2\sqrt{gL^3}) \tag{9c}$$

The values for the c_i in Table 1 are computed from Equations (9a, b, c) and numbers in the Value column. The resulting Froude number is Fr = 0.28191 ±0.00029 (±0.10 %) at the 95 % confidence limit with a cover factor k = 2. The length in the Froude number calculation is the length between perpendiculars from Longo and Stern (1998, 2005). In this example, the dominant uncertainty term is the velocity $V = 1.5410 \pm 0.0015$ m/s (±0.10 %). The contribution of g and L to the uncertainty in Fr are negligible. The results in the last column of Table 1, c_iU, have also been confirmed by the finite difference in Equation (4).

Table 1: Froude number with uncertainty estimates

Nomenclature	Symbol	Units	Value	U	c_i	c_iU
Velocity	V	m/s	1.5410	0.0015	0.18294	0.00028
Length	L	m	3.048	0.0015	0.04625	0.00007
Local Gravity	g	m/s^2	9.8031	0.00010	0.01438	0.00000
Froude No.	Fr		0.28191	0.00029		

5.2 Total Resistance Coefficient Example

The purpose of this example is to demonstrate the calculations for the uncertainty of the model total resistance coefficient, C_T, with a single run of multiple test runs. The data reduction equation is

$$C_\mathrm{T} = 2\,R_\mathrm{T}/(\rho V^2 S) \quad (10)$$

where R_T is the total resistance in Newtons (N), ρ the density of water in kg/m^3, V the velocity in m/s, and S the wetted surface area in m^2.

The uncertainty in resistance is initially from the calibration results. The force dynamometer is calibrated by the ITTC calibration procedure ITTC (2017b). Usually, the dynamometer is calibrated by a weight set. Mass units are then converted to force units by the equation from ITTC (2017b) and ASTM (2018)

$$F = mg(1 - \rho_\mathrm{a}/\rho_\mathrm{w}) \quad (11)$$

where m is the mass, g is local acceleration of gravity, ρ_a is air density, and ρ_w is the density of the weight. A calibration stand may include levers for increasing the force, in which case the force multiplier should be included in the above equation and the uncertainty estimates.

The last term of Equation (11) is an air buoyancy correction. Local gravity can differ from standard gravity, 9.80665 m/s^2, on the order of 0.1 %, and the buoyancy correction is typically 0.017 %. Mass sets commonly applied to force calibrations have a tolerance specification on the order of ±0.01 %.

Usually, the calibration uncertainty of the force dynamometer consists of three elements.

- Uncertainty by Type A evaluation from the time series for each data point when the data are collected by a computer system
- Uncertainty from the NMI calibration certificate for the weight set
- Uncertainty in the 95 % prediction limit from the linear regression analysis over the calibration range of the force dynamometer.

The uncertainty from the curve fit is typically the dominant term in the dynamometer calibration. For this analysis, a value of 0.11 % is applied from Longo and Stern (1998, 2005).

The water density is calculated from the ITTC water density procedure, ITTC (2011) with the water temperature. From Longo and

Stern (1998), the water temperature was 23.5 ±0.2 C. The resulting water density is then 994.422 ±0.048 kg/m^3. The carriage speed and its uncertainty is the same as reported in Table 1.

The uncertainty in wetted surface area, S, was estimated by Longo and Stern (1998) as 0.50 %. The estimate was based on the combined uncertainty in the hull form and displacement. With current methods, the surface area can be computed from an electronic engineering drawing of the hull, and the uncertainty estimated from laser measurements of the hull. With current methods, the uncertainty in the wetted surface area is likely less than reported by Longo and Stern (1998). The uncertainty in the various elements of the measurements are summarized in Table 2.

Table 2: Resistance coefficient with uncertainty estimates

Nomenclature	Symbol	Units	Value	U	c_i	c_iU
Velocity	V	m/s	1.541	0.0015	0.00591	0.000009
Resistance	R_T	N	7.3928	0.0082	0.00062	0.000005
Wetted Surface	S	m^2	1.3707	0.0069	0.00332	0.000023
Temperature	T	C	23.5	0.20		
Density	ρ	kg/m^3	997.4216	0.048	0.00000	0.000000
Coefficient	C_T		0.004554	0.000025		

The four sensitivity coefficients in Table 2 are computed from Equations (3) and (10) as follows:

$$c_R = \partial C_T/\partial R_T = 2/(\rho V^2 S) \quad (12a)$$

$$c_\rho = \partial C_T/\partial \rho = -2R_T/(\rho^2 V^2 S) \quad (12b)$$

$$c_V = \partial C_T/\partial V = -4R_T/(\rho V^3 S) \quad (12c)$$

$$c_S = \partial C_T/\partial S = -2R_T/(\rho V^2 S^2) \quad (12d)$$

The values for the c_i in Table 2 are computed from Equations (12a, b, c, d) and numbers in the Value column. The resulting total resistance coefficient is then C_T = 0.004554 ±0.000025 (±0.55 %) at the 95 % confidence limit with a cover factor k = 2. In this example, the dominant uncertainty term is the wetted surface area S = 1.3707 ±0.0015 m^2 (±0.50 %) with a secondary influence of velocity, V. The contribution of the water density to the uncertainty is negligible. The results in the last column of Table 2, c_iU, have also been confirmed by the finite difference in Equation (4).

Results of multiple test runs are summarized in Table 3 from Longo and Stern (1998, 2005). Test #11 is the result from Table 2. For this example, the uncertainty for each test is assumed the same. The uncertainty should be slightly different as the velocity for each test will not be the same. The standard deviation of R_T from the time series during a single run contributed to the uncertainty by the Type A evaluation in addition to the calibration uncertainty of the dynamometer. As the table indicates, the uncertainty from the repeatability is larger than the uncertainty for each test. In this case, the result is C_T = 0.004554 ±0.000027 (±0.60 %) with a coverage factor of 2.18 from the Student-t distribution at the 95 % confidence limit. By comparison, the relative uncertainty for test #11 was ±0.55 %.

The previous uncertainty estimate is for the mean value for that series of tests at the 95 % confidence interval. However, if C_T is to be applied to some future value of resistance, the uncertainty should be based on the 95 % prediction limit. The expanded uncertainty at the 95 % prediction limit from Devore (2008) is

$$U = t_{0.025,n-1} s \sqrt{1 + 1/n} \approx 2s \quad (13)$$

For the previous example, the result is C_T = 0.004554 ±0.000042 (±0.93 %) with a coverage factor of 2.18 at the 95 % prediction limit.

Table 3: Model results for multiple runs

Test #	C_T	U_B
1	0.004548	0.000025
2	0.004567	0.000025
3	0.004563	0.000025
4	0.004588	0.000025
5	0.004526	0.000025
6	0.004543	0.000025
7	0.004517	0.000025
8	0.004568	0.000025
9	0.004545	0.000025
10	0.004553	0.000025
11	0.004554	0.000025
12	0.004567	0.000025
13	0.004561	0.000025
Average	0.004554	0.000025
Std Dev	0.000019	
k	2.18	
U_A	0.000011	
U_c	0.000027	
U_c (%)	0.60	

6. SUMMARY

This procedure provides the elements for the basic understanding of practical uncertainty analysis. Examples of Froude number, *Fr*, and total resistance coefficient, C_T, are provided for an actual resistance test by Longo and Stern (1998, 2005). The estimates for uncertainty are based upon current methods. The examples demonstrate the importance of uncertainty analysis. In both cases, one variable dominates the uncertainty estimate, which is typical of uncertainty analysis results. For *Fr*, the dominant term is velocity, *V*, and for C_T the wetted surface area, *S*.

An analytical method is described for the computation of the sensitivity coefficient from Equation (3). Checking the results by a central finite difference calculation is recommended via Equation (4), which has been added to this revision. For processes more complicated than *Fr* and C_T, the finite difference method should be applied.

In the previous version of this procedure, a factor of ½ was omitted in Equations (9b, c) for the sensitivity coefficients for Froude number. However, the values in Table 1 were correct.

An uncertainty estimate may be increased by repeat tests. For the 13 repeat runs of C_T in Table 3, the uncertainty estimate for the average from the repeat tests was higher at ±0.60 % than estimate for a single run at ±0.55 %. Typically, an uncertainty will be higher from repeat tests. A test at a representative test condition be should repeated at least 10 times for a better estimate of the uncertainty. The standard deviation from such a test provides data for an uncertainty estimate at both the confidence and prediction limits.

Other documentation should be reviewed for additional details. The following is a summary of recommended sources:

- ITTC Procedure 7.5-02-01-01 contains detailed information on uncertainty analysis applicable to naval hydrodynamics. It is an abbreviated version of the GUM JCGM (2008).
- ITTC Procedure 7.5-01-03-01 describes the methodology for instrument calibration and related uncertainty estimate. Typically, an uncertainty in calibration consists of three elements: NMI traceability of the calibration reference device, Type A evaluation from the standard deviation of the time series when the data are collected by a computer system, and prediction of limit of the curve fit from calibration theory. For low-noise instrument and a sufficiently accurate reference device, most of the uncertainty is in the curve fit, which is computed from a 95 % prediction limit theory for calibration.
- ITTC Procedure 7.5-02-02-02.1 describes a resistance test in a towing tank, which was part on an ITTC inter-laboratory test of CEHIPAR Model 2716 with length of 5.72 m, which was manufactured in Spain at Canal de Esperiencias Hidronámicas de El Prado (CEHIPAR).
- ITTC Procedure 7.5-02-01-03 contains information on the properties of freshwater and seawater with uncertainty estimates.

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