ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

CFD User's Guide

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CFD Verification

1. PURPOSE OF PROCEDURE

The Purpose of the procedure is to ensure to get high Quality CFD solutions.

2. CFD VERIFICATION

2.1 ASME guidelines

Procedures following ASME guidelines 2-5, verification analysis refers to documentation of order-of-accuracy, effects of artificial dissipation, grid dependence, and iterative convergence for a given application. Since the flows of interest here are steady, guideline 6 does not apply.

2.1.1 Guideline 2

Requires that the method be at least second-order accurate in space. Order-of-accuracy is an important property that describes the rate at which the method should converge, as the grid is refined, to the exact numerical solution. The order may, for some methods, be expressed for each term in the governing equations (i.e., term-by-term as suggested by guideline 1) or be determined a posteriori through a careful grid study. The former approach, which for finite-difference methods is derived from the leading truncated terms in the Taylor-series approximation of the partial derivatives, suffers from several major deficiencies: not all methods are amenable to such analysis (e.g., finite-analytic discretization); it is overly optimistic for problems using non-orthogonal and stretched grids; and for mixed-order methods, it fails to determine the spatial variation of order due to the changing balance between inertia, viscous, pressure, and turbulent forces. In contrast, the latter approach, which is subsequently discussed, gives the actual overall order-of-accuracy and is not method specific.

In its most general form, spatial overall order-of-accuracy may be expressed as a three-dimensional quantity (i.e., in each of the co-ordinate directions p_X, p_Y, p_Z) for both point wise (e.g., individual velocity profiles, pressure distributions, wave elevations) and integral (e.g., resistance coefficients) variables. To determine order, solutions on seven grids must be obtained and be in the asymptotic range. The grids are generated using grid doubling (or halving) independently in each direction and the solutions on each may be referred to as $f_j^{\xi^i}$ (i.e., $f_1^{\xi}, f_2^{\xi}, f_3^{\xi}, f_1^{\eta}, f_2^{\eta}, f_3^{\eta}, f_1^{\zeta}, f_2^{\zeta}, f_3^{\zeta}$) where j =1,2,3 corresponds to fine-, medium-, and coarse-grid solutions, respectively. Note, for convenience, $f_1^{\xi}, f_1^{\eta}, f_1^{\zeta}{}_1$ correspond to the same solution. The asymptotic range is achieved when the solutions are grid convergent such that the grid convergence parameter, defined as the relative change between grids

$$\varepsilon_{12}^{\xi^i} = \left(f_1^{\xi^i} - f_2^{\xi^i} \right) / \, f_1^{\xi^i} \quad (2.1)$$

is monotonically decreasing at the rate r^p (i.e., $\varepsilon_{23}^{\xi^i} \approx r^p \varepsilon_{12}^{\xi^i}$), where r is the grid-refinement ratio and p is the order-of-accuracy. Each solution may be expressed in a three-dimensional Taylor-series expansion. If the grids are analytical (i.e., transformation metrics are identical at common points between coarse, medium and

fine grids), the seven Taylor series may be manipulated to derive analytical expressions for order accuracy

$$p^{\xi^i} = \frac{\ln\left[\frac{\left(f_2^{\xi^i} - f_3^{\xi^i}\right)}{\left(f_1^{\xi^i} - f_2^{\xi^i}\right)}\right]}{\ln(2)} = \frac{\ln\left[\frac{\varepsilon_{23}^{\xi^i}}{\varepsilon_{12}^{\xi^i}}\frac{f_1^{\xi^i}}{f_2^{\xi^i}}\right]}{\ln(2)} \qquad (2.2)$$

If it is further assumed, as is commonly done in practice, that the order is uniform in all directions, only three solutions on grids doubled (or halved) in each direction are required to determine p from (2.2). A grid-doubling approach has the additional benefit of enabling the use of Richardson extrapolation to produce a high-order solution benchmark (order 3p on coarse grid)

$$\begin{aligned} f_2^{2p} &= f_1\left(1+\frac{1}{r^p-1}\right) - f_2\left(\frac{1}{r^p-1}\right) \\ f_3^{2p} &= f_2\left(1+\frac{1}{r^p-1}\right) - f_3\left(\frac{1}{r^p-1}\right) \\ f_3^{3p} &= f_2^{2p}\left(1+\frac{1}{r^p-1}\right) - f_3^{2p}\left(\frac{1}{r^p-1}\right) \end{aligned} \qquad (2.3)$$

The benchmark can then be used for determining the magnitude of numerical error relative to the benchmark and for evaluating grid dependence. The primary difficulty in determining order-of-accuracy is the requirement of achieving the asymptotic range for all solutions. For RANS methods, this is due to having to resolve a range of length scales (i.e., viscous sub-layer, turbulent boundary-layer, bow, shoulder, and transom waves, etc.) on a highly stretched, non-orthogonal grid and the effect of varying the grid number by a factor of 4 in each direction.

2.1.2 Guideline 3

Requires that inherent or explicit artificial dissipation must be assessed and minimized. Inherent dissipation arises due to truncation errors in the discretization scheme and is represented by even-ordered terms on the right-hand side of the modified equation (i.e., the actual equation solved by the difference scheme including the terms which contribute to the dissipation and dispersion errors). Since inherent dissipation is a function of discretization and grid size, it can only be minimized through grid convergence studies (Guideline 4). If the discretization scheme is amenable to von Neumann analysis, and derivation of the modified equation, the inherent dissipation and its functional relationship to grid size and time step may be quantified. Unfortunately, due to nonlinearities, three-dimensionality, boundary conditions, and multi-level schemes of practical Navier-Stokes methods, application of these analysis methods are difficult, and for some solution schemes may not be possible (e.g., finite element and finite analytic schemes). Explicit dissipation, on the other hand, is purposely added to some methods to damp oscillations and maintain stability. As such, minimization of errors due to excessive dissipation may be quantified through a parametric study wherein solution convergence and/or sensitivity with respect to the magnitude of the dissipation is established.

2.1.3 Guideline 4

Requires that grid independence or convergence be established. To rigorously meet this requirement, all grid parameters which affect the solution should be independently studied. Typical grid parameters include: grid type (C-type, H-type, O-type); number of points; clustering near walls and leading and trailing edges; aspect ratio; location of exit and outflow boundaries;

and minimum-spacing requirements for turbulence models. For each, grid convergence is indicated by small and monotonically decreasing ↝ (2.1). The magnitude of ↝ provides a measure of the grid convergence and, as shown in (2.2), the convergence of ↝ is related to the order-of-accuracy. Similarly, as discussed with regard to order-of-accuracy, three-dimensional and/or averaged e may be calculated for both pointwise and integral values. For three-dimensional ↝ seven grids are required to independently determine $\varepsilon_{12}^{\xi i}, \varepsilon_{23}^{\xi i}$. Here however, a grid doubling approach is not required. Instead, non-integer refinement ($1 < r < 2$) may be used. Unfortunately, as r approaches 1, ↝ will become arbitrarily small and as such loses validity in assessing grid dependence. Therefore, a grid convergence index (*GCI*) (Roache, 1994a), which is a better measure of change between solutions and defined as

$$GCI_{12}^{\xi i} = 3\left|\varepsilon_{12}^{\xi i}\right| / \left(r^{p^{\xi i}} - 1\right) \tag{2.4}$$

should be used. Note that the value 3 in (2.4) comes from simply making second-order methods with grid doubling the standard (i.e., for r = 2 and p = 2, *GCI* = ↝). For independent coordinate refinement, the total *GCI* can be determined by simply adding the contribution from each direction

$$GCI_{12} = GCI_{12}^{\xi} + GCI_{12}^{\eta} + GCI_{12}^{\zeta} \tag{2.5}$$

As a minimum requirement and an alternative to the three dimensional *GCI*, the averaged *GCI* may be determined with three solutions on grids which have been refined simultaneously in all directions. If the grids have a different r in each direction, a conservative *GCI* should be based on the smallest directional r. Finally, by calculating *GCI* for both pointwise and integral quantities, the grid convergence for each variable and region of the flow may be assessed. The primary difficulty in displaying grid convergence is in obtaining solutions in the asymptotic range. Although the *GCI* requires order-of-accuracy, either a p may be assumed if the order is not known, or ↝ may be used instead of *GCI* with the understanding that non-integer refinement directly affects the magnitude of ↝. Finally, the practicability of three dimensional *GCI* and a 7-grid convergence study has yet to be evaluated.

2.1.4 Guideline 5

Requires that iterative convergence must be addressed. Since direct solution of the linear equations that result from discretization is prohibitive, iterative solution techniques are used, in general, in most implicit CFD methods. Convergence error is due to stopping the iteration process and is defined as

$$\varepsilon_{it}^{n} = \phi^{n} - \tilde{\phi} \tag{2.6}$$

where ϕ_2^n is the numerical solution at the nth iteration and $\tilde{\phi}$ is the exact numerical solution to the discretized equations. Theoretically, iteration should continue until ε_{it3}^{n} is equal to machine zero. However, in practice, grid complexity (i.e., amount of stretching and non-orthogonality) affects the rate of convergence such that for realistic applications, driving ε_{it4}^{n} to machine zero is not possible. Therefore, minimization of convergence errors requires that appropriate measures of convergence be used and estimation of the uncertainty created by stopping. Convergence may be assessed using two methods: residuals defined as the difference between iterations or the imbalance of the discretized equations with the current-iteration $\phi^{n}{}_{2}$. Based upon either of these methods, the iteration process is stopped when the residuals/imbalance reach an

acceptably small value (i.e., a convergence criterion). Also, since the magnitude of the residuals is influenced by time step, under-relaxation, and initial conditions, it is often required that the residuals also drop a specified number of orders-of-magnitude. To estimate the uncertainty created by stopping the iteration process, two approaches are used. First, the convergence history of the solution variables (e.g., resistance, surface pressure, friction velocity) can be studied to show that for the given residual status, the solution is either invariant with further iteration or contains persistent oscillations. Second, equation imbalance may be used as a direct indication of the iteration error ε_{it}^n, since it can easily be shown that since $\tilde{\phi}$ exactly satisfies the discretized equations, the imbalance with ϕ^n is the same as ε_{it}^n back-substituted into the discretized equations.

2.1.5 Guideline 6

Requires that for transient calculations, phase error must be assessed and minimized. As mentioned in discussion of Guideline 3, von Neumann analysis and derivation of the modified equation may be used to determine the magnitude and terms contributing to the dispersion, or phase, error. Also, as previously discussed, these methods are best applied to simple finite-difference schemes and that for practical methods and problems, grid dependence and parametric studies are typically used to quantify phase error in conjunction with benchmark data.

In conclusion, it should be emphasized that the purpose of validation analysis is to assure high-quality solutions through estimation of uncertainty. Karniadakis (1995) has proposed a "numerical error bar" method which includes all contributions to the overall solution uncertainty. Based upon the present discussion, the total error bar would be comprised of modelling (i.e., turbulence, free-surface, and boundary-condition models) and numerical (i.e., iterative convergence, grid convergence, temporal discretization, and artificial dissipation) errors and each component would be assessed through the appropriate validation and/or verification analysis.

2.2 Implementation Recommendations

Implementation recommendations are divided into 5 steps.

2.2.1 Grid design and identification of important parameters.

For surface-ship and submarine flows, the important parameters include: grid number (for both RANS and free-surface grids); bow, stern, centreplane, free-surface, and sonar-dome leading- and trailing-edge clustering; location of inlet, outer, and outflow boundaries; and minimum-Y^+ studies. Previous experience is often useful in limiting the scope of parameter variation.

2.2.2 Convergence studies.

For each grid parameter, a 3-grid refinement study should be conducted. Iterative convergence must be demonstrated and uncertainty established for all solutions through the use of residuals and convergence behaviour of the solution. Pressure and wall-shear stress on the body surface and wake-centreline velocity should be plotted over the last 1000 or so iterations. Based upon both integral (i.e., resistance coefficients) and point wise (i.e., surface pressure, wall-shear stress, and selected boundary-layer profiles) quantities, determine grid convergence by calculating *e* and/or GCI.

2.2.3 Artificial dissipation.

If explicit artificial dissipation is used, show convergence of solutions, and/or stability limits, with respect to magnitude. Minimally select three values of dissipation and calculate the change between solutions in a fashion similar to calculating the grid convergence parameter δ.

2.2.4 Establish uncertainties and assemble numerical error bar.

Based upon steps 1-3, establish uncertainty due to iterative convergence, grid convergence, and artificial dissipation.

2.2.5 Order-of-accuracy and Richardson-extrapolated benchmark.

If grid doubling was not used in the grid-convergence studies, a new set of grids and solutions for either the three-dimensional (7 grids) or averaged (3 grids) order-of-accuracy determination must be generated. If all solutions are in the asymptotic range, calculate order using (2.1) for integral and pointwise quantities. With the true order-of-accuracy, recalculate the *GCI's*. Finally, calculate high-order benchmark using Richardson extrapolation (2.2) and determine grid dependence for both integral and pointwise quantities through comparison to the benchmark.

In conclusion, steps 1-4 are required to establish uncertainty, which should be quoted, but due to time and cost constraints, the scope of the uncertainty analysis may be limited. Step 5 is also useful since p is needed for calculation of CGI, Richardson extrapolation provides a high-order benchmark, and, maybe most importantly, knowledge of true order-of-accuracy on stretched and non-orthogonal grids should be useful in guiding future CFD developments.

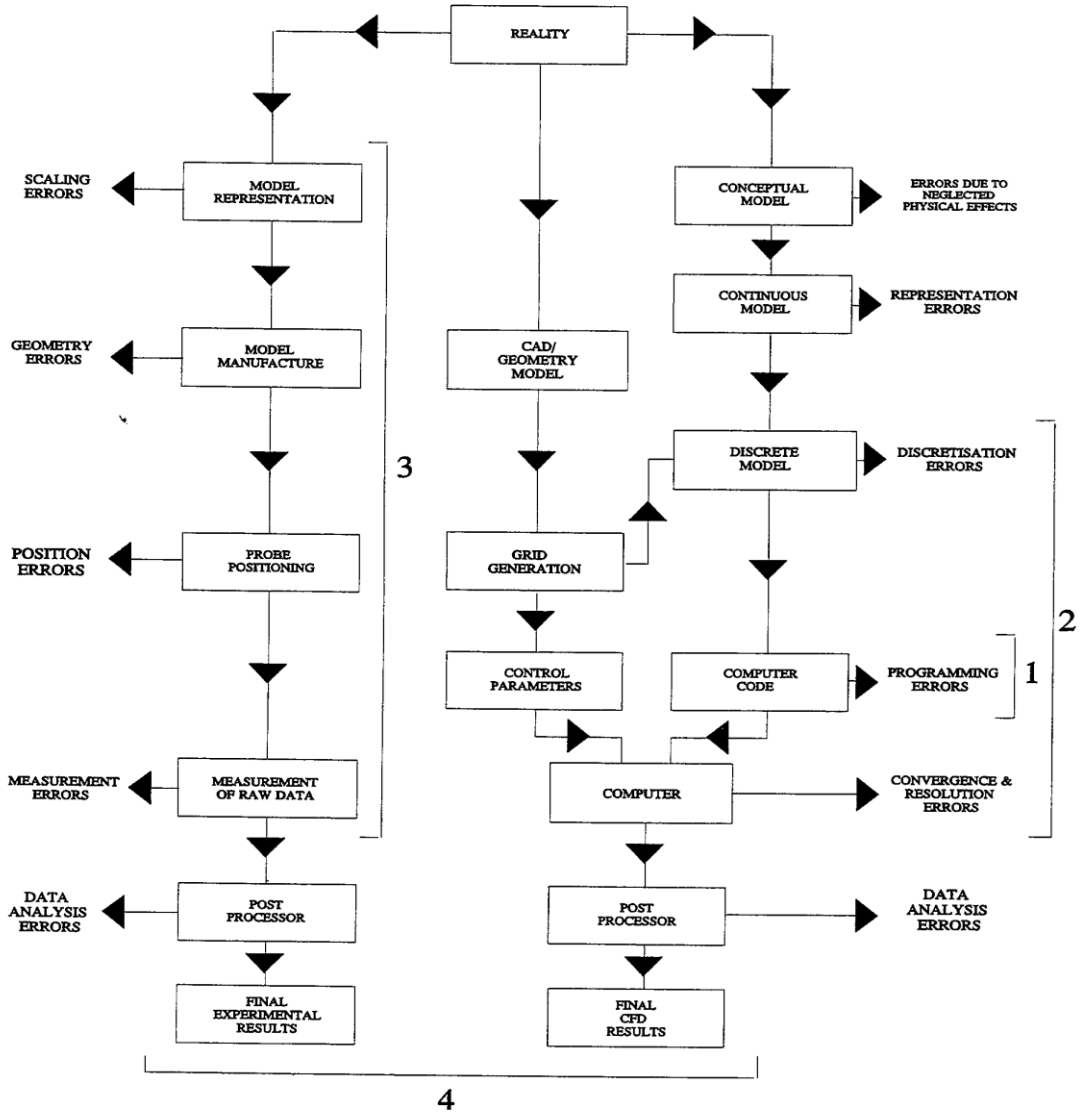

Figure 1 – CFD Validation