ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

Predicting the Occurrence and Magnitude of Parametric Rolling

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Updated / Edited by	Approved
Stability in Waves Committee of 28th ITTC	28th ITTC 2017
Date 05/2017	Date 09/2017

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Predicting the Occurrence and Magnitude of Parametric Rolling

1. PURPOSE OF GUIDELINE

This guideline describes the numerical methods for predicting the occurrence and magnitude of parametric rolling. It also highlights the limitations of these methods. For experimental methods investigating parametric rolling, the ITTC recommended procedure for model tests on intact stability (ITTC-Recommended Procedure 7.5-02-07-04.1) should be followed.

2. INTRODUCTION

The problem of parametric rolling of ships has been recognised for more than half a century. The fundamental dynamics that create this behaviour is considered in our days as reasonably clarified. Large as well as smaller ships have been investigated on the basis of theory and experiment, by and large for a following seas situation. In this state, it is easier to satisfy one of the necessary conditions namely that the frequency of encounter with waves whose length is similar or larger than the ship length is comparable to twice the roll natural frequency of ship. However, a recent accident with a post-Panamax containership in predominantly head seas, which led to extreme roll angles and accelerations with substantial loss and damage to containers stowed on the deck, was also attributed to parametric rolling.

Model tests and full scale observations have shown that parametric rolling can occur not only in long-crested (longitudinal) head and following seas, but also at slightly oblique heading angles with and without directional wave energy spreading. The physical phenomenon is based on successive alterations of the restoring lever between crests and troughs, exhibited by many ships in steep longitudinal waves. These set up a mechanism of internal (parametric) excitation in roll. There is a clear analogy with a simple oscillator governed by the so-called Mathieu equation with damping. If it is needed to understand the detail of the physical background of parametric rolling, IMO Document (Physical background of parametric roll, SDC3/WP.5) should be followed.

Under the important issues, such as the development of effective criteria for the prevention of parametric rolling by design, the assessment of the effects of coupling with other motions, and the derivation of optimal experimental / numerical procedures for safety assessment in a realistic sea, it is premature to aim at immediate development of a definitive procedure for conducting model experiments and numerical simulations in order to assess a particular hull form for parametric rolling. The current procedure represents progress towards this goal and it should be refined as more knowledge becomes available.

3. PREDICTION OF OCCURRENCE OF PARAMETRIC ROLLING IN REGULAR WAVES

Before treating the equations of motion that take into account parametric excitation and roll response, some essential elements of roll motion are reviewed first. At a fundamental level, the equation of linear rolling motion is that of an excited rotational oscillator:

$$\ddot{\phi} + 2\zeta\omega_{\phi}\dot{\phi} + \omega_{\phi}^{2}\phi = M_{x}(t)$$

where,

ϕ is the roll angle,

$\zeta = B/\{2\omega_\phi \cdot (I_x + J_x)\}$
is the roll damping ratio,

B is the ship's dimensional damping

$\omega_\phi = \sqrt{m\,g \cdot GM/(I_x + J_x)}$
is the roll natural frequency,

m is ship mass,

g is the acceleration of gravity,

I_x is the mass moment of inertia in roll and

J_x is the hydrodynamic roll moment of inertia. Finally,

$M_x(t)$ symbolises a moment representing the direct roll excitation.

Parametric rolling typically occurs in various combinations of ship speed and wave frequency, provided that the resulting frequency of encounter is near to (2/*n*) times the natural frequency, where *n* is any integer. The practical relevance of the *n*=1 scenario ("principal resonance", $\omega_e = 2\omega_\phi$) is well established for ships. The *n*=2 scenario ("fundamental resonance") is also believed to be of interest although with a lower probability of occurrence in a seaway.

The build-up of parametric rolling requires a threshold wave height in addition to fulfilment of the above condition of frequencies. The minimum wave height is determined in principle by two factors: the degree of fluctuation of roll restoring due to wave passage, and the ship's roll damping which is speed dependent. The damping is a key design parameter for the avoidance of parametric rolling. None of the current state-of-the-art computational programs can claim to calculate the roll damping accurately for any given vessel including all roll damping devices.

The restoring moment variation may be estimated based on balancing the vessel in the undisturbed wave at different roll angles and positions in the wave. It is noted, however, that effects related to forward speed and disturbed waves may influence the roll restoring moment. Longitudinal waves having a length of the order of the ship length will typically lead to the largest fluctuations of the roll restoring moment.

For the prediction of parametric rolling due to principal resonance the following simple rule may be applied, which is based on consideration of the asymptotic stability of the upright state of a ship in longitudinal waves (Francescutto *et al.* (2004); Spyrou, (2005)).

If $\overline{GM}$ varies on the wave between $\overline{GM}_{\min}$ and $\overline{GM}_{\max}$; and the scaled amplitude of variation of metacentric height, defined as follows:

$$h = \frac{\overline{GM}_{\max} - \overline{GM}_{\min}}{2\overline{GM}}$$

where $\overline{GM}$ is the mean metacentric height of the ship for the considered regular wave, exceeds 4 times roll damping ratio; then the occurrence of parametric rolling is possible.

This variation of $\overline{GM}$ can be reflected in the equation of roll motion as follows:

$$\ddot{\varphi} + 2\zeta\,\omega_\varphi\,\dot{\varphi} + \omega_\varphi^{\,2}\{1 - h\cdot\cos(\omega_e t - \varepsilon)\}\varphi = M_x(t)$$

where

ω_e is the encounter frequency and

ε is an appropriate phase difference.

The value of ω_ϕ here could be different from its calm-water value and obtained as follows:

$$\omega_\varphi = \sqrt{m\,g\cdot\overline{GM}/(I_x + J_x)}$$

In an accurate head or following sea, the direct wave excitation at the right-hand-side becomes zero.

In the vicinity of exact principal resonance, the following expression may be used for the threshold level of the scaled $\overline{GM}$ fluctuation (Francescutto *et al.* (2004); Spyrou, (2005)):

$$h = \sqrt{\left(2 - \frac{\omega_e^2}{2\,\omega_\phi^2}\right)^2 + 4 \cdot \zeta^2 \frac{\omega_e^2}{\omega_\phi^2}}$$

The further that we move upwards, in terms of *h*, from the above boundary curve, the quicker the build-up of parametric rolling becomes.

4. PREDICTION OF AMPLITUDE OF PARAMETRIC ROLLING IN REGULAR WAVES

If the probability of parametric rolling is "controlled" yet not completely eliminated by design, it is essential to ensure that the amplitude of parametric rolling oscillation that might be generated in an extreme seaway is kept small. For a typical ship with nonlinear restoring and damping, parametric rolling is usually bounded, reaching a steady-state amplitude that is proportional to the square root of the amplitude of restoring fluctuation. In this section, nonlinear formula of restoring moment first, and nonlinear formula of damping are shown.

More specifically, if the amplitude of roll is small to moderate, and depending on the detailed shape of the restoring lever, a third-order polynomial could represent reasonably the exact shape of the initial part of the lever. The nonlinear equation of roll in a longitudinal sea would be like:

$$\ddot{\varphi} + 2\zeta\,\omega_\varphi\,\dot{\varphi} + \omega_\varphi^2\left[1 - h\cos\left(\omega_e\,t\right)\right]\varphi - c_3\omega_\varphi^2\,\varphi^3 = 0$$

Then, the following expression could be used for predicting the steady roll amplitude *A* in the vicinity of principal resonance (Spyrou, (2005)):

$$A^2 = \frac{4}{3c_3}\left[\left(1 - \frac{1}{a}\right) \mp \sqrt{\frac{h^2}{4} - \frac{4\zeta^2}{a}}\right]$$

In the above $a = 4\,\omega_\phi^2 / \omega_e^2$. The ⊏ sign indicates the possibility of multiple, stable/unstable parametric roll oscillations coexisting for the same frequency ratio. In general, the larger *A* corresponds to the stable solution which is the realizable amplitude.

If the amplitude of parametric roll is moderate to large, a fifth order polynomial is likely to be required. In such a case the following expression of the amplitude could be useful (Spyrou, (2005)):

$$A^2 = -\frac{3c_3}{5c_5} \pm \sqrt{\left(\frac{3c_3}{5c_5}\right)^2 - \frac{8}{5c_5}\left(-1 + \frac{1}{a} \pm \sqrt{\frac{h^2}{4} - \frac{4\zeta^2}{a}}\right)}$$

In the above, c_3, c_5 are nonlinear stiffness coefficients, corresponding respectively to the third and fifth order restoring terms, according to the following roll equation:

$$\ddot{\varphi} + 2\zeta\omega_\varphi\,\dot{\varphi} + \omega_\varphi^2\left[1 - h\cos\left(\omega_e\,t\right)\right]\varphi - c_3\,\omega_\varphi^2\,\varphi^3 - c_5\,\omega_\varphi^2\,\varphi^5 = 0$$

The given formula for the amplitude of parametric roll can be used if the restoring curve is initially hardening (c_3<0) and then softening (c_5>0). It can be deduced that up to 4 coexisting stable/unstable solutions become possible for some values of the frequency ratio. For $a \geq 1$, the solution of the smallest amplitude is stable and stability alternates as we move towards the coexisting higher roll amplitudes, for the same value of the frequency ratio a. For $a<1$ the principle is the same, however we should start with an unstable solution.

In usual case, roll damping is also nonlinear, and linear and cubic damping coefficient are used. By including a cubic damping term $\delta\omega_\varphi\dot{\phi}^3$ the equation of amplitude of roll motion with a third-order restoring term is modified.

$$\ddot{\varphi}+2\zeta\,\omega_\varphi\,\dot{\varphi}+\delta\omega_\varphi\varphi^3+\omega_\varphi^2\left[1-h\cos\left(\omega_e t\right)\right]\varphi-c_3\omega_\varphi^2\,\varphi^3=0$$

The steady amplitude should become

$$A^2=\frac{4}{3c_3}\left[\left(1-\frac{1}{a}\right)\mp\sqrt{\frac{h^2}{4}-\left(\frac{2\zeta}{\sqrt{a}}+\frac{3\delta}{4a^{3/2}}\right)^2}\right]$$

The nonlinear damping is much smaller than the coefficient of the linear. Quantitative assessment of the various contributions to the amplitude A suggested that the effects of nonlinear damping to the reduction of A is much lower than that of the linear (Spyrou, 2005).

Parametric rolling could be characterised as "severe" if the steady amplitude is higher than 15 deg. The above simple expressions can be used in order to check whether this limit is exceeded. In case the GZ curve is more complicated and it is applicable for large amplitude rolling including capsizing, the following formula can be alternatively used to estimate the amplitude (Hashimoto et al., 2004).

$$\left\{\frac{\pi^2\omega_e\left(3A^2\omega_e^{\,2}\varsigma\omega_\varphi+8\delta\omega_\varphi\right)}{8\left(2\pi-A^2\right)\omega_\varphi^{\,2}}\right\}^2+\left\{\frac{\pi^2\left(\omega_e^{\,2}-4\omega_\varphi^{\,2}\right)}{2\left(\pi^2-A^2\right)\omega_\varphi^{\,2}}+\frac{2\pi^2\omega_\varphi^{\,2}\sum_{k=1}^{n}\frac{1}{2^{2k+1}}\frac{(2k+1)!}{(k+1)!(k+1)!}c_{2k+1}A^{2k}}{\left(\pi^2-A^2\right)\omega_\varphi^{\,2}}\right\}^2=h^2$$

where the roll equation is the following

$$\ddot{\varphi}+2\zeta\omega_\varphi\,\dot{\varphi}+\delta\omega_\varphi\dot{\varphi}^3+\omega_\varphi^2\left[1-h\cos\left(\omega_e t\right)\right]\left\{1-\left(\frac{\varphi}{\pi}\right)^2\right\}\varphi+-c_3\,\omega_\varphi^2\,\varphi^3-c_5\,\omega_\varphi^2\,\varphi^5\cdots-c_{2n+1}\omega_\varphi^{\,2}\varphi^{2n+1}=0$$

The c_{2n+1} coefficients of GZ curve can be determined with a least square fit to the GZ curve and the ζ and δ coefficients of damping can also be determined by following the ITTC recommended procedures for numerical estimation of roll damping (ITTC-Recommended Procedures 7.5-02-07-04.5).

The above mentioned equations of amplitude of parametric rolling are obtained from an averaging method assuming a period even if roll-restring moment is strongly nonlinear for roll amplitude. The results are reasonably good agreement with the results obtained by numerical simulation with a Poincaré mapping technique, which is a method able to identify bifurcation structures of nonlinear problem, in longitudinal seas. In quartering seas, however, the averaging method used here on its own is limited when identifying the complicated bifurcation structures of roll motions, e.g., chaos, that is far from the period-2 orbit. (Hashimoto *et al.*(2004)).

5. EVALUATION OF MODELLING METHODS

For the prediction of parametric roll in following seas, a single-degree-of-freedom mathematical model (and the associated simple analytical formulae proposed in the above) offers accurate prediction of the critical wave or of the required roll damping to prevent occurrence of parametric rolling, if the mathematical model considers hydrodynamic forces appropriately. Hashimoto *et al.*(2004) investigated a wave effect on roll restoring moment in following seas. And it is confirmed that the Froude-Krylov assumption generally overestimates the amplitude of roll moment variation and cannot explain the wavelength effect as well. The average of roll moment variation depends on nominal Froude number, i.e. advanced speed. Since the difference between measurements and the calculation based on the Froude-Krylov assumption is significant, the amplitude and the average of roll moment variation are recommended to be estimated not from the Froude-Krylov calculation, but from captive model tests.

For the prediction of parametric roll in head seas, heave and pitch motions should be considered because they are coupled with the roll motion and cross-coupling radiation forces are induced when the roll angle is not zero. Use of a coupled model of roll, heave and pitch is recommended for dynamic analysis of parametric roll in head seas, particularly when ship speed is not zero. Several numerical models for parametric rolling were developed and some of them were validated with their model experiments in head and following waves (Reed, 2011). These models are mostly based on coupled heave-pitch-roll models using simultaneous nonlinear differential equations and the hydrodynamic coefficients used in the equations are calculated with potential theories and empirical viscos force estimation. Nowadays CFD (Computational Fluid Dynamics) calculation could be an alternative in the estimation of roll damping coefficients (e.g. Report of the Stability in Wave Committee, 2014). Time variation of hydrodynamic coefficients of radiation and diffraction forces is recommended to include because they change significantly when large amplitude parametric roll happens. The nonlinear radiation and diffraction forces, as well as the nonlinear Froude-Krylov force, are important elements for the prediction of parametric roll in head seas.

For the prediction of parametric roll in oblique seas, above-mentioned heave-pitch-roll coupling motion models may not be sufficient, because manoeuvring motion including rudder actions are unavoidable in this situation. Umeda et al.(2015) attempted to validate a 5 degrees-of-freedom numerical simulation, taking low-speed manoeuvring model, in oblique seas by comparing with measured results conducted in a seakeeping and manoeuvring model basin. The numerical model, including the nonlinear Froude-Krylov force, radiation and diffraction forces calculated as a function of roll angle and manoeuvring forces, can predict experimental results qualitatively. Further improvement is expected for quantitative prediction of parametric roll in quartering seas.

6. PARAMETRIC ROLLING IN AN IRREGULAR SEAWAY

The variation of $\overline{GM}$ that is theoretically tolerable for a regular wave environment, in the sense that it does not give rise to parametric rolling, should be distinguished from the $\overline{GM}$ variation that could be practically permissible in a realistic seaway. The limited (rather than infinite) run length of critical wave groups and the possibly low probability to be encountered by a ship, mean that if the standard approach based on the deterministic criterion of asymptotic stability is applied, the ensuing design requirements may become unnecessarily stringent.

Stochastic parametric oscillations have been investigated extensively. However, as there are various definitions of stochastic stability even for a linear one-degree-of-freedom dynamical system, there has been no conclusion yet about the most suitable one for parametric rolling. In general, the development of a criterion that is suitable for a realistic sea is still open.

The narrower the sea spectrum, the more prominent become the wave groupiness, and the higher the probability of exceeding the threshold. In a following sea, the $\overline{GM}$ fluctuation could show, for an observer moving with the ship, a very narrow spectrum, even if the sea spectrum is quite wide (e.g. Bretschneider spectrum). This can result in a concentration of wave energy within a very narrow range of encounter frequencies, for certain heading angles in following/ quartering seas. Subsequently, a ship could experience a dangerous, regular-like parametric excitation if the frequency condition associated with parametric roll is approximately satisfied and if the associated waves are of critical height and length. On the other hand, in a head sea the $\overline{GM}$ fluctuation could show a wide spectrum even in moderately narrow spectra (e.g. JONSWAP). Model tests have shown that parametric rolling can be excited even in head seas very quickly during the passage of a wave group with critical characteristics.

Questions have been raised recently about the assumption of ergodicity of parametric roll during model testing in a wave basin and for numerical simulations of finite duration. This could create some uncertainty for current experimental or numerical assessment methods if these are based on finite temporal averages of roll motion. Whilst the issue is still unsettled, the fact that parametric rolling is essentially nonlinear and also that it is clearly related to wave groupiness, call for developing a new approach that goes beyond the standard linear seakeeping analysis.

For this reason, it is desirable to obtain not only a temporal average from a single run but also an ensemble average from multiple runs.

Integration of the above assessment of parametric rolling within a risk assessment procedure entails specification of the consequences. As it well-known, risk is defined as the product of the probability of occurrence times the consequences. It is not appropriate, even in a liberal sense, to consider as identical the "probability of parametric roll" with the "risk of parametric roll". The consequences of parametric roll could vary qualitatively as well as quantitatively. Ship capsize should be considered as rare whilst more likely should be damage to the cargo. This damage should be quantified in order to determine the risk Furthermore, the susceptibility to parametric rolling should be distinguished from the probability of parametric rolling; because in addition to how prone a ship is to parametric rolling, the latter contains also the probability of encounter of critical environmental conditions. At this stage, it is untimely to propose a detailed guidance for carrying out risk assessment of parametric rolling as the field is currently under development.

7. REFERENCES

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