ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

Uncertainty Analysis for a Wave Energy Converter

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-07	Loads and Responses
7.5-02-07-03	Ocean Engineering
7.5-02-07-03.12	Uncertainty Analysis for a Wave Energy Converter

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Uncertainty Analysis for a Wave Energy Converter

1. PURPOSE OF PROCEDURE

This purpose of this document is to provide guidance for ITTC members to perform Uncertainty Analysis (UA) of Wave Energy Converters (WECs) following the ITTC Guideline 7.5-02-07-03.7, “Wave Energy Converter Model Test Experiments”.

This guideline is based on ISO (1995) and in line with other ITTC uncertainty analysis (UA) procedures such as ITTC Recommended Procedures and Guidelines (7.5-02-01-01, “Guide to the Expression of Uncertainty in Experimental Hydrodynamics” and 7.5-02-06-05, “Uncertainty analysis for free running manoeuvring model tests”) that are recommended to maritime experimental facilities. The main purpose and measurement variables of WEC tank tests depend on the targeted Technology Readiness Levels (TRLs) of the device. Model tests of WECs have some differences from tests of other offshore structures including several additional challenges as listed in the ITTC Guideline 7.5-02-07-03.7.

2. INTRODUCTION

Testing the performance of WECs requires a detailed understanding of the device interactions with ocean waves. For instance, Figure 1 illustrates a simplified wave energy conversion chain where it can be seen that a part of the incoming wave energy (E_I) is reflected (E_R) and/or transmitted (E_T) due to the wave–device hydrodynamic interactions, with the rest of this energy representing the energy absorbed by the device (E_A). This absorbed energy is the maximum energy that can be further converted into useful electricity (E_E) after considering the radiated energy (E_D) due to device motions and/or the chamber’s free surface oscillations in case of oscillating water columns, and the energy losses (E_L) in forms of viscous, turbulences and mechanical losses.

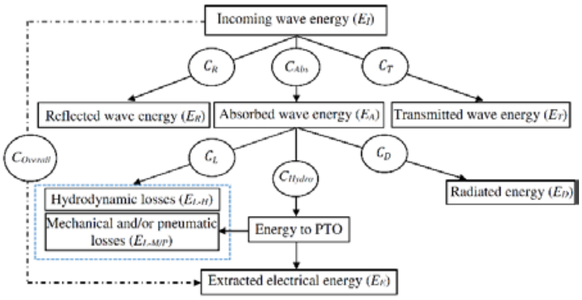


Figure 1: Energy conversion chain in a WEC

The conversion efficiency between each energy component is represented by a coefficient (C). According to the energy conservation principal, an energy balance model can be written as in Equation (1). For early TRLs (1–4) (Nielsen, 2002) where the full Power Take–Off (PTO) system is not included but simulated by the use of orifices, mesh or damper, the extracted mechanical energy (E_E) is not directly measured, instead it is estimated based on experimental measurements and power train efficiency assumptions. As a result, quantifying the uncertainties in the output energy/power requires a methodology that considers the different uncertainties in each relevant measured parameter. A part of the extracted mechanical energy can be converted to electrical energy via for example utilizing an electric generator that adds more uncertainty in the final output energy; however, this uncertainty is not included in the current procedure.

$$E_I = E_R + E_T + E_D + E_L + E_E \tag{1}$$

As reported in ITTC Guideline 7.5-02-07-03.7 model tests of WECs can have different purposes. Once the objectives of the test have been identified, it is possible to select appropriate uncertainty analysis and design the experiment methodologies. Although every test procedure is individual, the adoption of the general outline test process formulated by AIAA (1999) and adopted by the ITTC Procedures 7.5-02-01-01 provides a means of introduction and integration of uncertainty assessment into each phase of the experimental process, with appropriate decision points and reporting. It stresses the importance of uncertainty analysis as “the foundation of all [towing] tank experiments”, and that UA should be performed both prior and post experimental work as part of the planning and designing of the test as well as the post–processing of the results.

3. UNCERTAINTY CLASSIFICA-TION

A measurement is a process of estimating the value of a quantity. Every measurement is accompanied by error(s). This error is defined as the difference between the measured value and the ‘true value’, and can be decomposed into bias error and precision error as illustrated in Figure 2.

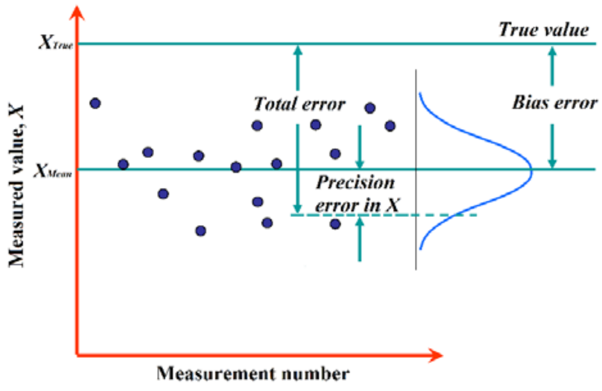


Figure 2: Bias and precision errors in a measurement.

Instead of using bias and precision errors, ASME PTC 19.1 (2005) uses the terms systematic and random errors. The former refers to the portion of total error that remains constant in repeated measurements of the true value throughout the test, while the latter describes the portion of the total error that varies in repeated measurements and causes scattering in the measured data.

The true value of a measured quantity is usually unknown. Therefore, the objective of uncertainty analysis is to estimate reasonable limits that combines the bias and precision errors and to construct an uncertainty interval within which the true value of the measured variable can be expected to lie within a chosen level of confidence (Forgach, 2002).

ISO (1995) classifies uncertainties into three categories: Standard Uncertainty, Combined Uncertainty, and Expanded Uncertainty.

3.1 Standard uncertainty (u_S)

ASME PTC 19.1 (2005) utilizes two major classifications for measurement uncertainties, random and systematic uncertainties, and describes the limits to which random and systematic errors may lie within a chosen level of confidence. On the other side, according to ISO (1995), the standard uncertainty of the result of a measurement can be grouped into two types, Type A uncertainties and Type B uncertainties, depending on the method and information available for estimation of uncertainty. Type A uncertainty components are obtained using a method based on statistical analysis of a series of observations/repeats, whereas Type B uncertainty component is obtained by means other than repeated observations such as prior experience, professional judgements, manufacturers’ specifications and calibration of the sensors (ITTC Procedure 7.5-02-01-01).

Considering the Recommendation INC-1 (1980) that indicated that the term ‘systematic uncertainty’ can be misleading and should be avoided, the ISO classification will be considered in this document. Accordingly, the standard uncertainty (u_S) that combines both uncertainty types is given by Equation (2) as the root–sum–square (RSS) combination of Type A uncertainty (u_{S-A}) and Type B uncertainty (u_{S-B}).

$$u_S = \sqrt{\left(u_{S-A}\right)^2 + \left(u_{S-B}\right)^2} \tag{2}$$

3.2 Combined standard uncertainty (u_c)

The final result from an experiment is not always being measured, instead it is calculated from different measured parameters using a mathematical model. Consequently, quantifying the uncertainty in this result requires a methodology to combine the uncertainty associated with each parameter. In other words, the combined standard uncertainty (u_c) of the output variable is obtained from the uncertainties of a number of other quantities (input) considering that the quantities are either correlated (dependent) or not (independent).

For example, considering a quantity of interest (Y) defined in Equation (3) (called Data Reduction Equation, DRE) as a function (f) of other measured quantities ($X_1, X_2, \ldots\ldots, X_N$), the general equation for the combined standard uncertainty in Y is given in Equation (4). It is based on a first-order Taylor series approximation of the measurement equation of quantity $Y = f(X_1, X_2, \ldots\ldots, X_N)$ and its estimated value (y) (ISO, 1995):

$$Y = f\left(X_1, X_2, \ldots\ldots, X_N\right) \tag{3}$$

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u_s^2(x_i) + \\ +2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right) u_s\left(x_i, x_j\right) \tag{4}$$

where u_c is the combined uncertainty of the Y estimated at $Y = y$; y is estimate of Y and calculated from Equation (3) at $X_i = x_i$; x_i is the estimate of X_i; $\frac{\partial f}{\partial x_i}$ is the partial derivative of f with respect to the X_i(commonly referred to as sensitivity coefficients or uncertainty magnification factors, UMFs) and evaluated at $X_i = x_i$; $u_s(x_i)$ is the standard uncertainty associated with the input x_i evaluated at $X_i = x_i$; and $u_s(x_i, x_j)$ is the estimated covariance associated with x_i and x_j. In cases of practical interest, Equation (4) can be reduced to a simple form by neglecting the second term assuming the different x_i to be independent to each other ($u_s(x_i, x_j) = 0$).

For example, the regular wave energy (E_I) per unit width given as a function of the incident wave height (H) and length (L) is:

$$E_I = \frac{1}{8}\rho g H^2 L \tag{5}$$

Considering the uncertainties associated with all the parameters in Equation (5) including the water density (ρ) in the testing facility and the gravitation acceleration (g), and that all parameters are independent, the DRE can be applied to calculate the uncertainty in the wave energy (u_{E_I}), giving:

$$u_c^2(E_I) = \left(\frac{\partial E_I}{\partial \rho}\right)^2 u_s^2(\rho) + \left(\frac{\partial E_I}{\partial g}\right)^2 u_s^2(g) + \\ +\left(\frac{\partial E_I}{\partial H}\right)^2 u_s^2(H)\left(\frac{\partial E_I}{\partial L}\right)^2 u_s^2(L) \tag{6}$$

The sensitivity coefficients for all parameters in Equation (6) are summarized in Table 1.

Table 1: Sensitivity coefficients for a regular wave energy

Parameter	Sensitivity coefficient
ρ	$\frac{\partial E_I}{\partial \rho}=\frac{1}{8}gH^2L$
g	$\frac{\partial E_I}{\partial g}=\frac{1}{8}\rho H^2L$
H	$\frac{\partial E_I}{\partial H}=\frac{1}{4}\rho gHL$
L	$\frac{\partial E_I}{\partial L}=\frac{1}{8}\rho gH^2$

The mean and standard uncertainty for the fresh water density are calculated based on ITTC Recommended Procedures 7.5-02-01-03, “Fresh Water and Seawater Properties” as 998.207 kg/m^3 ± 0.0105 kg/m^3 at a temperature of 20 ± 0.10 °C, while the mean and standard uncertainty for the gravitational acceleration according to ITTC Recommended Procedures and Guidelines 7.5-01-03-01, “Uncertainty Analysis, Instrument Calibration” are 9.80665 m/s^2 ± 0.0057 m/s^2 assuming rectangular/uniform distribution. On the other hand, uncertainties in wave height and length are estimated from Equation (2) considering both Type A and B uncertainties as described in 0.

It is not always possible to mathematically formulate the Data Reduction Equation. In that case, a proper numerical model can be employed to find a linear relation between each variable (input) in the DRE and the final output. This technique is extensively discussed in ITTC Procedure 7.5-02-06-05, but it is briefly explained in the following. In order to find the UMF of a certain input parameter, at least two simulations are required. The initial condition for the second simulation for the input parameter must be controlled such that a highly linear trend can be drawn. This can be achieved by carefully studying the relation between the input and output variables for a range of initial conditions such that a linear slope representing the UMF can be determined. The simulation model does not have to be very accurate, but it is important that the trend is correctly predicted.

3.3 Expanded uncertainty $(\boldsymbol{U})$

The combined standard uncertainty (u_c) maybe thought of as equivalent to ‘one standard deviation’, but we may wish to have an overall uncertainty stated at another level of confidence. From practical viewpoint, in experimental hydrodynamics and flow measurements, an interval with a level of confidence of 95% is justifiable (ITTC Procedure 7.5-02-01-01). Accordingly, the expanded uncertainty (U) with this confidence level requires scaling the combined standard uncertainty (u_c) using a coverage factor (k) as given in Equation (7).

$$U = ku_c \tag{7}$$

Usually a coverage factor k = 2.0 is used for a level of confidence of 95%. However, it is worth noting that this assumes a Gaussian distribution with at least 61 data sampling size (this provides a degree of freedom of 60). For a lower number of samples/repeats used to calculate a standard deviation, a student T–Distribution must be used to determine the coverage factor, which commonly is provided in T-Distribution tables.

4. EVALUATION OF STANDARD UNCERTAINTY

4.1 Evaluation of Type A uncertainty

When a set of several repeated readings has been taken, each individual observation is expected to have a different value from other observations due to the random variations of the influence quantities, or random effects. From these repeats, the standard uncertainty (u_{S-A}) is defined as the estimated standard deviation of the mean as in Equation (8).

$$u_{S-A} = \frac{s}{\sqrt{n}} \tag{8}$$

where s and n are the standard deviation and the number of repeated observations.

The standard deviation (s) of the n repeated readings is calculated from Equation (9).

$$s = \sqrt{\frac{\sum_{k=1}^{n} (q_k - \overline{q})^2}{n-1}} \tag{9}$$

where q_k is the k^{th} repeated reading and $\overline{q}$ is the mean value of the whole repeated readings as given in Equation (10):

$$\overline{q} = \frac{\sum_{k=1}^{n} q_k}{n} \tag{10}$$

Testing of WECs usually includes a large number of conditions, and therefore, it is not practicable to carry out multiple repeats for every experimental run. It may be more feasible to only select unique test conditions such as at device's resonance for which repeat runs should be undertaken so that Type A uncertainty can be estimated. Numbers of repeats should be as large as practicable, but this is subject to cost and schedule constrains. ITTC Procedure 7.5-02-01-01 stated that 10 repeats should provide a reasonable estimate of Type A uncertainty.

According to ISO (1995), the following conditions should be considered for experiment repeatability:

a) The same measurement procedure,
b) The same measuring instrument used under the same test "environmental" conditions,
c) The same location, laboratory, or field location
d) Repetition over a short period of time, roughly, tests are performed in the same day.

The repeated runs should include sequential (ITTC, 2011) and non–sequential repeats ITTC Recommended Procedures 7.5-02-07-03.1, "Testing and Extrapolation Methods Loads and Responses, Ocean Engineering Floating Offshore Platform Experiments".

4.2 Evaluation of Type B uncertainty

As mentioned in 1.1, Type B uncertainty is not based on statistical methods, but its evaluation is usually based on experience and judgment. Therefore, it heavily depends on considering all relevant information available, which may include (ISO, 1995):

- Previous measurement data;
- Experience with or general knowledge of the behaviour and properties of relevant materials and instruments;
- Manufacturer's specifications;
- Data provided in calibration and other certificate, which must be traceable to National Metrology Institutes (NMI);
- Uncertainties assigned to reference data taken from handbooks.

The proper use of the pool of available information for a Type B evaluation of standard uncertainty calls for insight based on experience and general knowledge, and is a skill that can be learned with practice. Type B uncertainty is usually evaluated based on information quoted in a handbook, manufacturer's specification, calibration certificate, etc. In this case, the standard uncertainty can be provided as a multiple of an estimated standard deviation or a confidence interval. Other means of obtaining a Type B uncertainty are by assuming the provided data follow a certain distribution (such as normal distribution), but when it is only possible to estimate bounds (upper and lower limits) for the measured quantity, and there is no specific knowledge about the possible values of this quantity within this interval/limits, one can only assume a uniform/rectangular distribution.

Testing WEC's includes utilizing different sensors such as wave probes, load cells, pressure transducers, motion tracking system, etc. There are elemental Type B uncertainties that are an inherent part of each sensor, calibration, the data acquisition system (DAS), processing and analysis. Uncertainty sources that are commonly provided by the manufacturer includes non–linearly, hysteresis, zero offset drift, non–repeatability, resolution, etc. Sensors calibration is mandatory for all instruments before being used in the experiment so that instrument's uncertainty can be characterised. However, the calibration process itself includes uncertainties. All calibration should be performed through either system calibration or end–to–end calibration with the same DAS and software as utilized during data collection. The calibration results should be reported so that new calibrations can be compared. Most instrumentation is highly linear; therefore, a linear fit of the calibrated data is usually applied, and the standard uncertainty is defined by the standard error of estimate (SEE) as in Equation (11). Further details with examples on linear and non–linear calibration curve fitting and uncertainties in mass used in calibrating load cells is provided in ITTC Procedure 7.5-01-03-01.

$$SEE = \sqrt{\frac{\left(y_j - \widehat{y_j}\right)^2}{M-2}} \tag{11}$$

where and M is the number of calibration samples/points and $y_j - \hat{y}_j$ is the difference between calibrated data point and the fitted value.

5. UNCERTAINTY ANALYSIS FOR DESIGN OF EXPERIMENT

Uncertainty analysis is necessary for planning an experiment, and/or improving the results of future experiments. The purpose of Design of Experiment (DoE) is to optimise in advance an experimental process in order to collect high quality data, which means minimizing as much as possible uncertainty sources. The flow chart in Figure 3 illustrates the required steps to test a WEC in a cost-effective way.

The chart in Figure 3 breaks down the test procedures into a series of steps that should be considered during testing WECs, especially small and medium scale (TRL1–TRL4) (Nielsen, 2002) considering the limited large tanks available for testing large scales.

In any test preparation, a pre–test uncertainty analysis should be performed during the planning and designing phases of the test with the same computer code applied during the test. This enables the identification of critical measurements that need to be measured more carefully and/or factors that may need to be repeated more than others to drive uncertainty to desired levels. This analysis includes primarily Type B uncertainties unless data are available from previous tests for an estimate of the Type A uncertainties. Selection of an instrument may involve economic trade-offs between cost and performance.

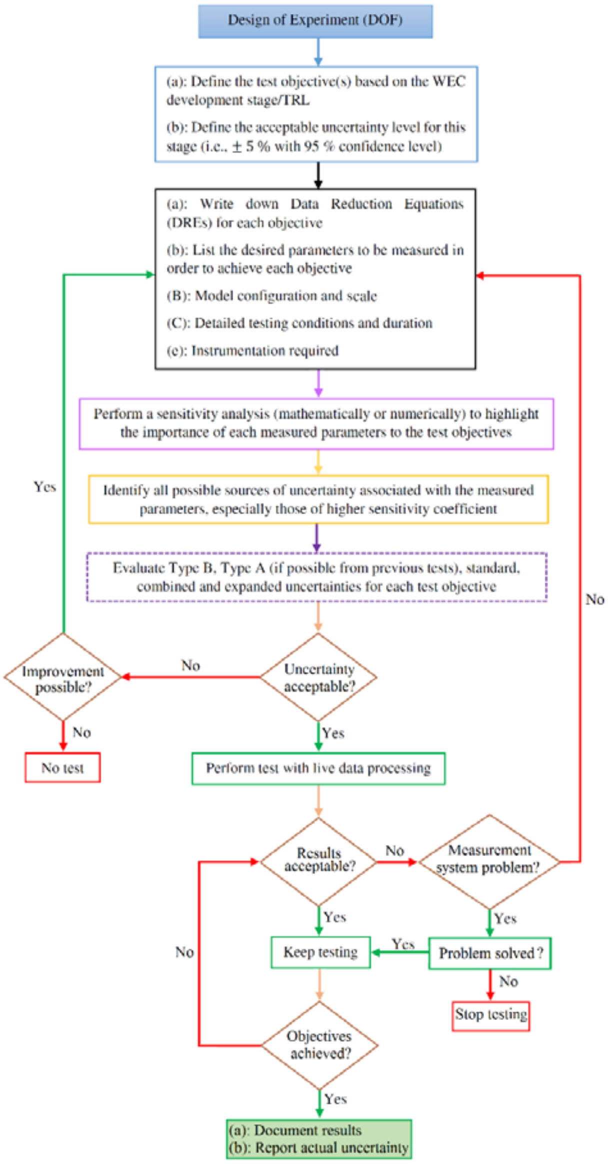


Figure 3: Flow chart of experimental process, indicating decision points and information sources. Adapted from ITTC Procedure 7.5-02-01-02.

Accordingly, in this phase, all elements of the Type B uncertainty should be applied. In particular, manufacturers' specifications may be included for an assessment identifying the quality of the instrumentation needed for acquisition of the desired experimental results. In some cases, an uncertainty analysis indicates that the desired results cannot be achieved and that the experiment should be abandoned.

6. LISTING AND DISCUSSION OF THE SOURCES OF UNCERTAINTY

An important step in the flow chart in Figure 3 is to define all possible uncertainty sources. Considering the different uncertainty sources provided in ITTC Procedure 7.5-02-06-05 for hydrodynamic experiments, the uncertainty sources that might be encountered when testing a WEC are listed below.

- Inaccuracy of WEC model characteristics including: geometry/dimensions, mass, centre of gravity, GM, draft, moment of inertia, model orientation to the incoming waves, especially for wave–direction dependent devices such as terminator and attenuator devices, mooring lines anchoring points and inclination angle in case of taut mooting with multiple lines.
- Undesired facility related hydrodynamic effects including discrepancy between nominal and measured wave characteristics. Often, the measured waves are different from the desired condition (input to wavemaker), especially the wave height. In addition, due to wave–wave interactions, the generated waves are not homogenous/consistent throughout the tank which increases the uncertainty in the measured waves and highlights the importance of properly identifying the testing area in the facility, as well as the exact deployment location of the device together with the measured wave characteristics at that location. ITTC Recommended Procedures and Guidelines 7.5-02-07-04.1, "Testing and Extrapolation Methods Loads

and Responses, Stability Model Tests on Intact Stability" states that wave elevation should be monitored at more than three locations covering the testing area with variations in wave height and period should be within 5% among the different measured positions. Furthermore, residual free surface oscillations in the testing facility (flume, tank or basin), especially if the waiting time between each runs is insufficient. This affects the initial conditions for the following run, which in turn influences the testing device dynamics. The waiting time depends mainly on the facility and the testing conditions, and it is recommended to be specified using previous testing experience within the same facility. In addition, the tank width and bottom profile may impact the collected data.

- Errors in PTO system control equipment parameters such as size of orifice, turbine propeller rate of rotations, copper loss (resistance in the electric circuit of the PTO system), etc.;
- Disturbance from test arrangement of the model such as using signal cables for wave probes and pressure sensors attached to the model as in floating Oscillating Water Columns (OWCs);
- Measurement inaccuracies due to calibration or improper installation of instruments such as misalignment in an Acoustic Doppler Velocimetry (ADV), laser sheet for Particle Image Velocimetry (PIV), laser displacement sensors and potentiometers, etc.

7. EXAMPLE OF UNCERTAINTY ANALYSIS APPLIED TO AN OWC TYPE WEC EXPERIMENTAL TEST

7.1 Introduction to the experiment

A 1:50 model–scale of a floating offshore OWC–WEC with dimensions illustrated (Figure 4) is considered in this section to quantify the uncertainties in the physical measurements performed in the 100 m long, 3.5 m wide, 1.5 m deep towing tank of the Australian Maritime College (AMC), University of Tasmania, Australia (see Figure 5). The tank is equipped with a flap–type wavemaker at one end and a wave–absorption at the other end.

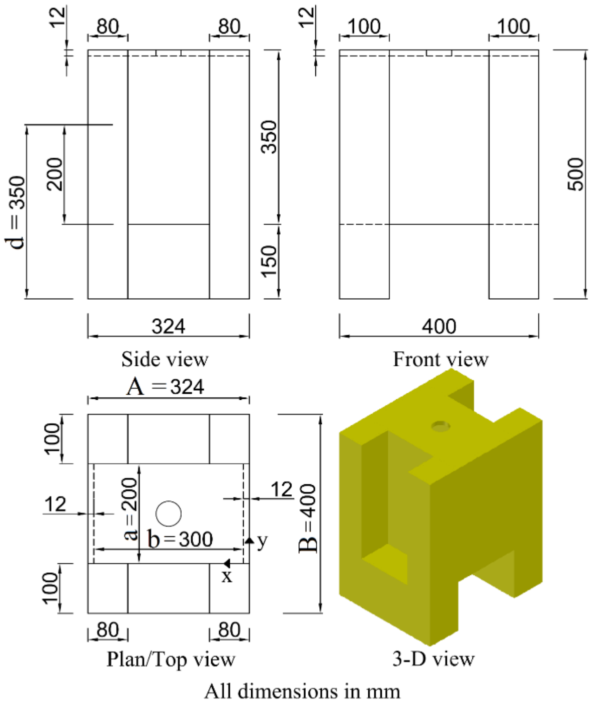


Figure 4: 1:50 offshore–stationary OWC dimensions

In order to monitor the wave envelope resulting from the wave–OWC hydrodynamic interactions, six custom made resistive–type wave probes WP (names as WP_0 – WP_5) were installed along the tank as shown in the experiment layout in Figure 6 (a). WP_0 measured the incident waves, WP_1 – WP_3 were used to resolve the incident and reflected waves (energy), WP_4 measured the waves (energy) transmitted on the model's leeside and WP_5 (phase WP) was employed to provide information regarding the incoming waves approaching the model's front wall.

Figure 5: A general view of the AMC towing tank showing the OWC model installed in the tank, looking towards the beach.

The OWC was fitted with three WPs: one at the centreline of OWC's front wall to measure the wave run–up (WP$_6$) and the other two (WP$_7$ – WP$_8$) installed inside the OWC's chamber for averaging the measured water level elevation (η) and the free surface vertical velocity ($d\eta/dt$). In addition, two pressure sensors, Honeywell–TruStability–001PD TSC Series (P$_1$ and P$_2$) for averaging the chamber's differential air pressure ($\Delta P(t)$), were installed on the OWC's top plate (see Figure 6(b)). Having defined the free surface vertical velocity and assuming incompressible air for the small scale used in the experiment, airflow rate ($q(t)$) can be calculated as in Equation (12) and then the time–averaged extracted pneumatic power (P_E) and the overall hydrodynamic non-dimensional capture width (ζ) are calculated from Equations (13) and (14), respectively.

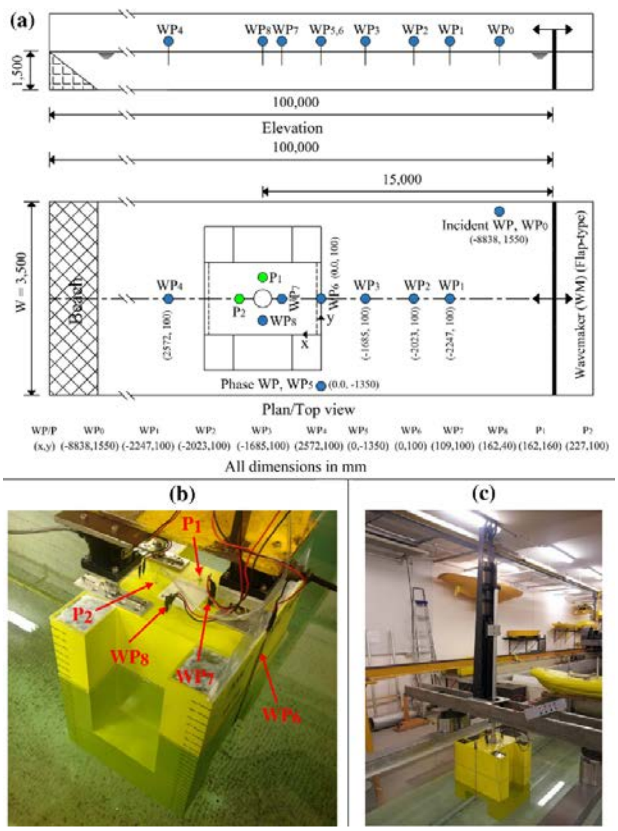


Figure 6: (a) Experiment layout (not to scale), (b) OWC wave probes and pressure sensors, and (c) OWC wave probes calibration procedure

$$q(t) = \frac{d\eta}{dt} ba \left[\frac{\mathrm{m}^3}{\mathrm{s}} \right] \tag{12}$$

where b and a are the chamber's length and width, respectively (see Figure 4).

$$P_E = \frac{1}{T} \int_0^T \Delta P(t) q(t) dt \, [\mathrm{W}] \tag{13}$$

where T is the wave period.

$$\zeta = \frac{P_E}{P_I a} [-] \tag{14}$$

where P_I is the incident wave energy flux (power) per unit width is defined as the product of the total (potential and kinetic) wave energy

(E_I) per unit ocean surface area and the group velocity (C_g) (Dalrymple and Dean, 1991):

$$P_I = \frac{1}{8}\rho g H^2 C_g \left[\frac{\mathrm{W}}{\mathrm{m}}\right] \tag{15}$$

The six wave probes along the tank were calibrated daily, whereas OWC wave probes and pressure transducers were calibrated before and after completing the experiment. All OWC wave probes were calibrated at the same time as illustrated in Figure 6 (C). All measurements were sampled at 200 Hz.

7.2 Measurement uncertainty analysis

The two uncertainty types described in 0 are calculated as follows:

7.2.1 Type B uncertainty

The calibration procedure for the pressure sensors and wave probes were performed by in-situ end-to-end calibration procedure with the same data acquisition system and software applied during the tests, which is advisable by ITTC Recommended Procedures and Guidelines 7.5-02-07-02.1, “Seakeeping Experiments”. This procedure characterises the sensor/instruments uncertainty as it includes as many of the possible Type B uncertainties in the calibration procedure so that details of uncertainty analysis of signal conditioning and data acquisition system is not necessary. Following the calibration process, the curve fitting’s standard Type B uncertainty (u_{S-B}) is estimated using the standard error of estimation (SEE) 1.5 given in Equation (16).

$$u_{S-B} = SEE = \sqrt{\frac{\left(y_j - \widehat{y_j}\right)^2}{M-2}} \tag{16}$$

where M is number of calibration samples/points, y_i and $\widehat{y_i}$ are the calibrated data point and the fitted value from the linear regression analysis, respectively.

7.2.2 Type A uncertainty

As discussed in 1.4, Type A uncertainty (u_{s-A}) depends on the experiment repeatability and this uncertainty is estimated as the standard deviation of the mean given by Equation (8). Examples of the experiment repeatability in time series are shown in Figure 7 for two tested conditions of H = 0.05 m, T = 1.2 s and H = 0.10 m, T = 1.2 s.

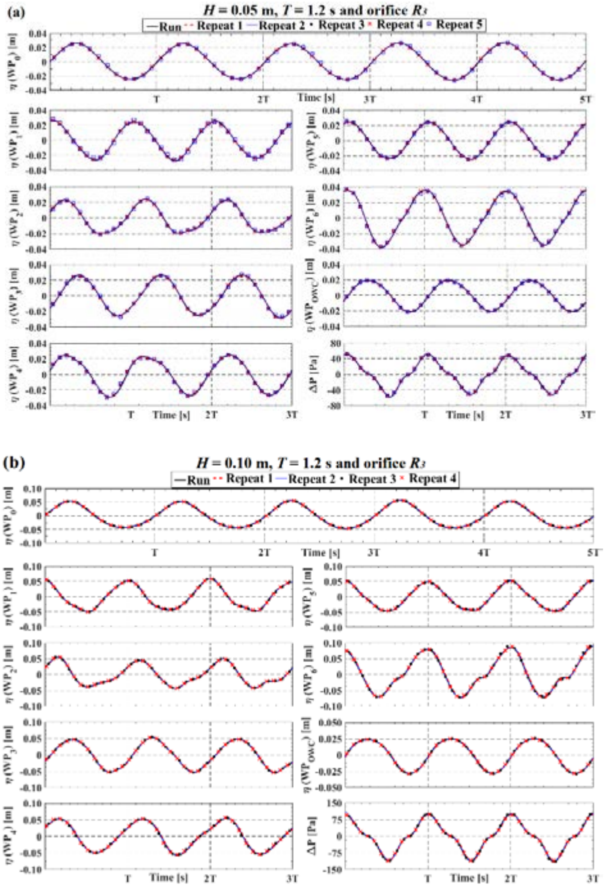


Figure 7: Experiment repeatability. (a) H = 0.05 m, T = 1.2 s, R3 and (b) H = 0.10 m, T = 1.2 s and orifice R3 (radius = 17.84 mm)

7.2.3 Standard uncertainty

After evaluating Type A and Type B uncertainties for each load cell component, the standard uncertainty (u_s) that combines both uncertainty types is calculated by (2).

7.2.4 Combined uncertainty

The chamber's differential air pressure is the average of two pressure sensors (P1 and P2). Similarly, the chamber's free surface oscillation is the average of WP7 and WP8. Accordingly, the combined standard uncertainties in air pressure (u_{S-P}) and free surface oscillation (u_{S-OWC}) are computed via the law of propagation of uncertainty described in 1.2 (and shown again below in Equation (17)) as given in Equations (18) and (19), respectively (ITTC Procedure 7.5-02-01-01):

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u_s^2(x_i) \qquad (17)$$

$$u_{S-P} = \sqrt{\left(\frac{u_{S-P1}}{2} \right)^2 + \left(\frac{u_{S-P2}}{2} \right)^2} \qquad (18)$$

where u_{S-P1} and u_{S-P2} are the standard uncertainty for pressure sensors P_1 and P_2, respectively.

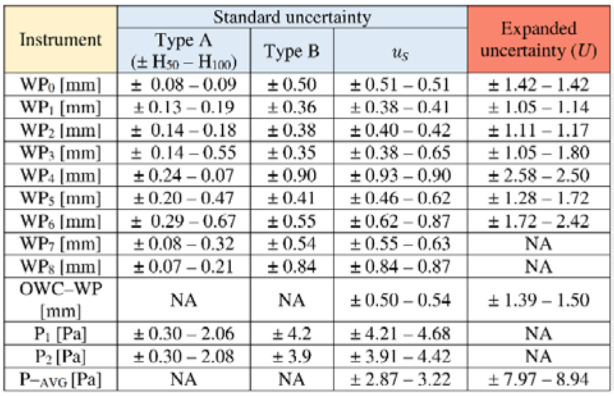
$$u_{S-OWC} = \sqrt{\left(\frac{u_{S-WP7}}{2} \right)^2 + \left(\frac{u_{S-WP8}}{2} \right)^2} \qquad (19)$$

where u_{S-WP7} and u_{S-WP8} are the standard uncertainty for WP_7 and WP_8, respectively.

7.2.5 Expanded uncertainty

The expanded uncertainties (U) with a level of confidence of 95% are summarized in Table 2, where H_{50} and H_{100} refer to the uncertainty for regular waves of height 50 and 100 mm, respectively. For the five runs considered in this example, a coverage factor k = 2.776 is applied based on T-Distribution tables.

Table 2: Experimental uncertainties



Instrument	Standard uncertainty			Expanded uncertainty (U)
	Type A (± H_{50} – H_{100})	Type B	u_S	
WP_0 [mm]	± 0.08 – 0.09	± 0.50	± 0.51 – 0.51	± 1.42 – 1.42
WP_1 [mm]	± 0.13 – 0.19	± 0.36	± 0.38 – 0.41	± 1.05 – 1.14
WP_2 [mm]	± 0.14 – 0.18	± 0.38	± 0.40 – 0.42	± 1.11 – 1.17
WP_3 [mm]	± 0.14 – 0.55	± 0.35	± 0.38 – 0.65	± 1.05 – 1.80
WP_4 [mm]	± 0.24 – 0.07	± 0.90	± 0.93 – 0.90	± 2.58 – 2.50
WP_5 [mm]	± 0.20 – 0.47	± 0.41	± 0.46 – 0.62	± 1.28 – 1.72
WP_6 [mm]	± 0.29 – 0.67	± 0.55	± 0.62 – 0.87	± 1.72 – 2.42
WP_7 [mm]	± 0.08 – 0.32	± 0.54	± 0.55 – 0.63	NA
WP_8 [mm]	± 0.07 – 0.21	± 0.84	± 0.84 – 0.87	NA
OWC–WP [mm]	NA	NA	± 0.50 – 0.54	± 1.39 – 1.50
P_1 [Pa]	± 0.30 – 2.06	± 4.2	± 4.21 – 4.68	NA
P_2 [Pa]	± 0.30 – 2.08	± 3.9	± 3.91 – 4.42	NA
P_{-AVG} [Pa]	NA	NA	± 2.87 – 3.22	± 7.97 – 8.94

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