ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Confidence Intervals for Significant Wave Height and Modal Period

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Confidence Intervals for Significant Wave Height and Modal Period

1. PURPOSE OF PROCEDURE

The purpose of this procedure is to formulate the process for characterizing the uncertainty of wave data resulting from random seas data collected at either model or full scale. Wave data is collected in both controlled and uncontrolled environments—typically in a controlled environment for model-scale data and in an uncontrolled environment for full-scale data. The contribution of uncertainty is characterized by a variance of the estimates.

This procedure only deals with the statistical uncertainty of the random wave data resulting from the finite size of the sample.

2. INTRODUCTION

Measured wave data are considered random numbers because the environment is intrinsically random and the sample size is finite. Random wave data are characterized in terms of significant wave height and modal period, there is uncertainty associated with each of these quantities.

The statistical uncertainty of significant wave height and modal period will be expressed in terms of confidence intervals for their estimates.

2.1 Statistical Uncertainty

Statistical uncertainty analysed in this procedure is a result of the finite size of the sample data set, making averages random. The assumption of a normal distribution for these averages is based on the Central Limit Theorem.

A normal distribution is defined by its mean value and variance. The mean value of the estimate approximately equals the estimate itself. The variance of the estimate is computed from the record time-series data. Thus, the uncertainty is quantified by the variance of the estimates (*i.e.*, mean and variance).

The calculation of the variance of an estimate has to account for the dependency of data points within each record, which are near to each other in time. Two different procedures are provided for significant wave height and one is presented for modal period.

3. SPECTRAL ANALYSIS

To analyze and verify the wave conditions, produced in a wave basin or observed at sea, spectral analysis techniques must be employed.

The wave data that is to be used for spectral analysis must be despiked and calibrated wave. The frequency spectrum of the waves is obtained by performing a Fourier transform on the raw wave data, producing the spectral estimate $\hat{S}(\omega)$ for the wave time series. The spectrum is formed applying no windowing or overlapping.

Throughout this document the estimate from the measured data will be denoted by a "hat". For example $\hat{S}(\omega)$ is the measured spectral estimate while $S(\omega)$ is the theoretical/target spectrum. Additionally, it should be noted that the spectral shape $S(\omega)$ can be related to the spectrum in Hz, $S_f(f)$ by the following relation:

$$S_f(f) = 2\pi\, S\left(\frac{\omega}{2\pi}\right)$$

The integral of the resulting frequency spectrum with respect to angular frequency is equal to the variance of data, or

$$\hat{\sigma}^2 = \int_0^\infty \hat{S}(\omega)\, d\omega$$

where $\hat{\sigma}$ is a standard deviation of the time history of wave elevation.

3.1 Significant Wave Height

As the wave data consists of a time series of random data, the significant wave height and its confidence limits can be derived directly from the time series using the techniques presented in ITTC Procedure N.N-NN-NN-NN, "Single Significant Amplitude and Confidence Intervals for Stochastic Processes," ITTC (2017), remembering that the significant wave height is four (4) time the standard deviation of the measured waves, rather than the factor of two (2) for single significant amplitude (SSA). Alternatively, the technique of Young (1986, 1995) can be used to obtain the significant wave height and its confidence bands directly from integrals of the spectrum. This will be described below.

In the approach of Young (1986, 1995), the variability in the spectral estimate $\hat{S}(\omega)$ can be improved by ensemble averaging. In ensemble averaging, the time series is subdivided into q individual time series. Spectral estimates are individually calculated for each of these smaller time series. The q estimates of the spectral ordinate at each frequency are averaged to obtain the final spectrum. The averaged values of the spectral ordinates follow a chi-squared distribution with $n = 2q$ degrees of freedom (Young, 1995). The $(1-\alpha)$ tolerance interval for the spectrum $\hat{S}(\omega)$ based on the estimate $S(\omega)$ is given by:

$$\frac{\chi^2_{n;\frac{\alpha}{2}}}{n} S(\omega) \le \hat{S}(\omega) \le \frac{\chi^2_{n;1-\frac{\alpha}{2}}}{n} S(\omega)$$

where $\chi^2_{n;\alpha}$ is the α point of the standard chi-square distribution where $\chi^2_{n;0} < \chi^2_{n;1}$. The confidence interval is similarly (Young, 1986, 1995):

$$\frac{n}{\chi^2_{n;1-\frac{\alpha}{2}}} \hat{S}(\omega) \le S(\omega) \le \frac{n}{\chi^2_{n;\frac{\alpha}{2}}} \hat{S}(\omega)$$

The significant wave height, H_s can be calculated from

$$H_s = 4\sigma = 4\sqrt{\int_0^\infty S(\omega)\, d\omega}$$

The estimate of the variance, $\hat{\sigma}^2$ determined from the spectrum follows a chi-square distribution with v degrees of freedom,

$$v = \frac{n\left[\int_0^\infty S_f(f)\right]^2}{\Delta f \int_0^\infty \left[S_f(f)\right]^2}$$

where Δf is the frequency discretization of the spectral estimate $\hat{S}_f(f)$ which is a function of sample length:

$$\Delta f = \frac{1}{T}$$

where T is the sample length. For the case where the theoretical spectrum $S_f(f)$ is evaluated at the same spectral ordinates as the target spectrum this is simplified to the following formula found in Young (1986, 1995):

$$\nu = \frac{n\left[\sum_{j=1}^{M} S_f\left(f_j\right)\right]^2}{\sum_{j=1}^{M}\left[S_f\left(f_j\right)\right]^2} = \frac{n\left[\sum_{j=1}^{M} S\left(\omega_j\right)\right]^2}{\sum_{j=1}^{M}\left[S\left(\omega_j\right)\right]^2}$$

Therefore the tolerance bands are:

$$\left[\frac{\chi^2_{\nu;\frac{\alpha}{2}}}{\nu}\right]^{\frac{1}{2}} H_s \le \hat{H}_s \le \left[\frac{\chi^2_{\nu;1-\frac{\alpha}{2}}}{\nu}\right]^{\frac{1}{2}} H_s$$

and the confidence bands for the spectrum estimate are (Young, 1986, 1995):

$$\left[\frac{\nu}{\chi^2_{\nu;1-\frac{\alpha}{2}}}\right]^{\frac{1}{2}} \hat{H}_s \le H_s \le \left[\frac{\nu}{\chi^2_{\nu;\frac{\alpha}{2}}}\right]^{\frac{1}{2}} \hat{H}_s$$

3.2 Modal Period

The modal period directly proportional to the inverse of the spectrally weighted peak frequency ($T = 1/f = 2\pi/\omega$), which can in turn be determined by the techniques presented in Young (1995).

The spectrally weighted peak frequency f_p is determined by using a weighted integral (Young, 1995):

$$f_p = \frac{\int_0^\infty f S_f^4\left(f\right) df}{\int_0^\infty S_f^4\left(f\right) df}$$

The 95% tolerance limits for $\hat{f}_p$ are:

$$\frac{a_l}{a_m} f_p \le \hat{f}_p < \frac{a_u}{a_m} f_p \qquad (1)$$

where a_l and a_u are factors defining the lower and upper 95% tolerance limits respectively, and a_m is a de-biasing coefficient that is a function of the particular spectral shape.

Following Kent & Lee (2016), Eq. (1) deviates from Young (1995) in that this formula is given as being for the *confidence* interval when it is actually for the *tolerance* interval, Eq. (1) has also been modified to take in to account the width of the bands rather than the absolute value by dividing by the de-biasing coefficient a_m. This error was confirmed by performing several Monte-Carlo simulations to recreate the results in Young (1995). The coefficients in Young (1995) are given as a function of n, γ and $\Delta f / f_p$, and for analysis of data are interpolated linearly. The γ value is determined by fitting a JONSWAP spectrum to the theoretical target spectrum. Similar to the significant wave height, the 95% confidence limits for the peak frequency, f_p are found to be:

$$\frac{a_m}{a_u} \hat{f}_p \le f_p < \frac{a_m}{a_l} \hat{f}_p$$

4. REFERENCES

ITTC, 2017, “Single Significant Amplitude and Confidence Intervals for Stochastic Processes,” ITTC—Recommended Procedures and Guidelines, N.N-NN-NN-NN, 15 p.

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