# ITTC Quality System Manual

# Recommended Procedures and Guidelines

**Guideline**

# Laboratory Modelling of Waves: regular, irregular and extreme events

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| Updated / Edited by | Approved |
|---|---|
| Specialist Committee on Modelling Environmental Conditions of 28th ITTC | 28th ITTC 2017 |
| Date 03/2017 | Date 09/2017 |

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# Laboratory Modelling of Waves: spectrum, extreme events and measurements

## 1. PURPOSE OF GUIDELINE

The purpose of this recommended guideline is to help laboratories in conducting tests on waves. It is not exhaustive, but it focus on some rather basic questions like linear and nonlinear waves, regular and irregular wave conditions as well as some more challenging problems like the generation of extreme waves in a tank.

The guideline is mostly based on the ITTC Reports on the subject and more specifically on ITTC (2002), SC Committee on Waves, ITTC (1999) SC Committee on Environmental Modelling and on ITTC (2017) SC Committee on Modelling of Environmental Conditions.

## 2. WAVE MODELLING AND QUALITY IN MODEL TESTING

### 2.1 Regular waves

Ideally, regular waves are periodic unidirectional progressive wave trains, with a single (monochromatic) basic harmonic.

#### 2.1.1 Linear waves

For most regular wave applications, the average wave height H and the average period T are of main interest. Amplitudes A, defined by H/2 or by crests AC and troughs AT, and the average steepness kA, are also used (k is the angular wave number).

Ideally, properties should be constant throughout time and in space, but in physical generation there is always a certain level of variation. Time windows for analysis are selected on the basis of criteria such as minimum variations, minimum transient effects in the model test set-up, or minimum reflections from the beach or from walls. Normally a minimum of 10 wave cycles is selected. Parameters are defined by a time-domain (zero crossing) approach or by a Fourier (harmonic) approach (mainly the basic harmonic). Simple RMS analysis of elevation records is also applied.

#### 2.1.2 Non-linear effects

Real water waves are not exactly linear. They are characterized by nonlinear effects which are of higher order with respect to the wave amplitude. The higher order solutions may be derived by using the Stokes's expansion as shown in Newman (1967) or Dean and Dalrymple (2000), or they may be derived by using fully nonlinear methods (Rienecker and Fenton, 1981). In a non-dimensional form, the wave amplitude is better represented by the wave steepness ε = kA. With increasing the wave steepness ε, the wave profile progressively deviates from the pure sinusoidal wave and are characterised by higher crests and shallower troughs (Toffoli et al., 2005). The asymmetry with respect to the horizontal axis (which is called vertical asymmetry) is generally referred to as skewness which is defined as (e.g. Babanin, 2011)

$$S_k = \frac{A_C}{A_T} - 1$$

and it is positive, unless some other phenomena, e.g. breaking, occur.

Nonlinear effects on wave height distributions are discussed in Tayfun and Fedele (2007). The nonlinear effects in the water waves and in particular the vertical asymmetry are also responsible for the wave drift.

Non-linear regular wave characteristics are defined by components at higher harmonics. The asymmetric wave geometry, with increased crests and associated local steepness, may have important consequences in practice, as it is for instance the case of stability tests. Recent studies also show that the bound harmonics are also responsible for the generation of rogue waves (Fedele et al. 2016).

To distinguish these 'real' non-linear effects in open-sea wave fields from 'parasitic' laboratory-defined ones, comparisons of laboratory generated wave profiles with theoretical/numerical reference models are helpful. On this regard, although referring to a two-dimensional wave system, Henderson et al. (2006) shows the relevance of accounting for nonlinear effects in the wave generation in order to achieve a wave pattern that propagates with time independent form.

#### 2.1.3 Finite water depth

Wave generation in water of finite depths introduces additional effects relative to that in deep water. Dispersion is depth-dependent, with shorter wavelengths and reduced speed in decreased depths (Newman, 1967; Toffoli et al., 2005). This may lead to spatial variations due to refraction effects unless the bottom is perfectly horizontal and flat. Fully nonlinear solutions for wave propagation over topography are provided in Kennedy and Fenton (1997).

Non-linear wave-wave interactions increase with reduced depth, with sharper peaks but also larger set down effects and corresponding return currents.

#### 2.1.4 Waves on currents

Theoretically, a perfectly steady current that is collinear with the waves slightly reduces the wave heights and increases the wavelength. Similarly, an opposing current increases the wave heights and reduces the wavelength. Normally the specified model waves are calibrated with the current on, so the changes in wave height are accounted for and embedded in the resulting wave field. Non-linear wave-current interaction effects influence the resulting hydrodynamic forces, such as wave drift damping (and corresponding modification in slow-drift excitation), wave-induced currents, wave kinematics and others of vessels and offshore structures.

#### 2.1.5 Deviations from ideal conditions

Ideally, a regular wave modelling would require a unidirectional periodic wave field with amplitude, period and direction constant throughout time and space. In practice, deviations from the ideal situation are observed, for various reasons, which are associated with wave maker, basin and wave absorbing devices.

Model testing procedures must take these effects into account, in one or several of the following ways: a) avoiding them, b) reducing them, c) documenting them and interpreting their effect on a vessel and an offshore structure responses.

For instance, in a two-dimensional wave tank three-dimensional waves, albeit of small amplitude, can be generated due to the interaction with the wave tank walls. The phenomenon is noticeable in case of single flap wave makers and in those cases they can be suppressed, or at least significantly reduced in a given range of wavelength, by using straighteners in the initial portion of the tank. Another option, which performs usually better, is to use controlled multi-segmented wave maker, with paddles of small size compared to the tank. The use of control system also allows to suppress other disturbances (e.g. reflection from the beach).

If the disturbances cannot be suppressed, their effects on the measurements can be reduced by choosing a proper combination of location and time window. In all cases, reflections or other disturbances have to be measured and documented.

Another important aspect concern the model scale. When reducing the scale of the problem, and thus the wavelength, the waves are more keen to develop naturally the modulational instability (e.g. Tulin and Waseda, 1999). The phenomenon is partly reduced by the increased role played by the viscous dissipation (Ma et al., 2012) but a careful check of accuracy and repeatability of the wave quality is needed when using relatively small scales.

## 2.2 Irregular waves

### 2.2.1 Wave spectra

Sea states are generally specified by the short-term variance spectrum *S(f)* or *S(ω)*, where *f* and *ω* are the frequency and the angular frequency respectively. Primary spectral parameters are the significant wave height $H_s$, defined as $H_s = H_{m0} = 4\sqrt{M_0}$ and a characteristic wave period, e.g. the peak period $T_p$ or the zero-crossing period $T_z$ defined from the spectrum as $T_z = T_{m02} 2\pi\sqrt{M_0/M_2}$, the mean wave period $T_1 = T_{m01} 2\pi\, M_0/M_1$ where $M_i$ is the *i*-th spectral moment (DNV, 2011) defined as

$$M_n = \int_0^\infty \varpi^n\, S(\varpi) d\varpi$$

$S(\varpi)$ being the power spectral density.

Many widely-used models for the spectrum of waves measured at a point (without regard to wave direction) are of the form

$$S(f) = \frac{A}{f^5}\exp(-B/f^4) \qquad (1)$$

(Bretschneider, 1959) where *f* is the frequency and *A* and *B* are constants. Among this class are those referred to as Pierson & Moskowitz (one- and two-parameter forms), ISSC, ITTC, and Liu. These presentations differ only with respect to the parameters that are used in determining *A* and *B*.

Other spectra include those related to the basic Bretschneider form, for cases where there is a limited fetch (JONSWAP), a finite water depth (TMA), or a combination of a known wind speed and limited fetch (Mitsuyasu, 1972). The specified duration of random simulations is important to achieve stationary irregular wave conditions: normally, a duration of 1 hour for seakeeping and 3 hours in offshore engineering is applied.

### 2.2.2 Bi-modal or multi-modal spectra

In some cases the spectrum is characterized by two (or more) energy peaks occurring at different frequencies. This is the case when a swell generated remotely, combines with a wind sea generated by a storm located closer to observation point.

Bi-modal spectra, that is, wind sea plus a swell, are now frequently specified thanks to the improved field data documentation. Bi-modal spectrum are generally built by combining different solutions. Normally it is specified as the sum of two unimodal spectra, or by an integrated formula, with a given set of parameters. Bi-modal spectrum which are more common in use in laboratory are those by Ochi and Hubble (1976) and Torsethaugen (1993). Each component is often modelled unidirectional, collinear or in different directions, while directional spreading is sometimes included.

2.2.3 Directional spreading

Most likely, the multi-modal sea is also multidirectional. Generally, the swell component is unidirectional with very little directional spreading whereas the wind sea may be characterized by a more substantial directional spreading with the central (peak) direction oriented with a certain angle to the swell. A proper account for all the peaks of the frequency spectrum as well as of the directions of the different wave systems is fundamental for an accurate prediction of the ship response.

For modelling purposes, the directional characteristics of waves are sometimes assumed to be uncoupled from their spectral properties, and then the spectrum of waves travelling within a given range of headings is taken to be some proportion of that measured at a point. On this basis, the directional spectrum is usually presented in the form

$$S(f,\theta) = S(f)G(\theta),(2)$$

where the spreading function $G$ depends only on the direction $\theta$. Its most common form is

$$G(\theta) = F(s)\cos^{2s}\left[(\theta-\theta_1)/2\right], \quad (3)$$

where $\theta_1$ is the predominant wave direction, and $s$ is an index that determines the width of the directional spread. In other forms of $G(\theta)$, the power $2s$ is replaced by $s$, or the argument of the cosine may omit the factor $1/2$. In another approach it can be expressed just in terms of its angular harmonics (see e.g. Frigaard et al., 1997) for further details. The function

$$F(s) = \frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}, \quad (4)$$

ensures that the total variance of the directional spectrum $S(f,\theta)$ is the same as that of the point spectrum $S(f)$.

A more general and detailed discussion of the laboratory modelling of multi-modal and multi-directional wave spectra is provided in the Guideline 7.5-02-07-01.2.

## 2.3 Extreme events

2.3.1 Deterministic generation of extreme events

There is evidence that extreme events may be responsible for accidents on ships and offshore structures. Although there is not yet a consensus about the probability of occurrence and the extreme waves (also referred to as rogue or freak waves) are not included in the classification society rules and offshore stand-ards, in the last decades there has been a grow-ing interest towards the understanding of such events as well as in their reproduction in labor-atories. A review of some research work done on the subject is provided in the DNV-GL position paper (DNV-GL, 2015).

Although there is a debate about the physics behind the occurrence of rogue waves (Fedele et al., 2016), a rather reliable technique to generate rogue waves in a specific location and time in the tank is proposed in Chabchoub et al. (2012). The technique, which is so far developed for unidirectional waves only, is based on a weakly nonlinear formulation and exploits some solutions of the nonlinear Schrödinger equation to form a rogue wave. In particular the so called Peregrine breather solution is considered. The local free surface elevation is expressed by

$$q\,\eta(x,t) = Re\left\{q_p(x,t)\exp\left[i(k_0x-\omega_0t+\varphi)\right]\right\}$$

where $q(x,t)$ is the equation of the envelope of the Peregrine breather:

$$q_p(x,t) = a_0 \exp\left(-\frac{ik_0^2 a_0^2 \omega_0}{2} t\right) \times \left(1 - \frac{4(1 - ik_0^2 a_0^2 \omega_0 t)}{1 + \left[2\sqrt{2}\, k_0^2 a_0 (x - c_g t)\right]^2 + k_0^4 a_0^4 \omega_0^2 t^2}\right)$$

In the above equations, $a_0$, $\omega_0$ and $k_0$ indicate the amplitude, frequency and wave number and $c_g$ is the group velocity which can be estimated as half the phase velocity.

The above solution is valid for deep water. More recently, the solution has been generalized to finite water depth. The solution, which is provided in Onorato et al. (2013), is basically the same but the equation of the envelops is a little more complicated:

$$q_p(x,t) = a_0 \exp\left(-i\beta a_0^2 t\right) \times \left(\frac{4\alpha(1 - i2\beta a_0^2 t)}{\alpha + \alpha\left(2\beta a_0^2 t\right)^2 + 2\beta a_0^2 x^2} - 1\right)$$

where:

$$\alpha = \frac{1}{2\omega_0}\left(c_g^2 - gh\,\mathrm{sech}^2[k_0 h]\left(1 - k_0 h \tanh^2[k_0 h]\right)\right)$$

and

$$\beta = \frac{\omega_0 k_0^2}{2}\left(\frac{8 + \cosh[4k_0 h] - 2\tanh[k_0 h]^2}{8\sinh[k_0 h]^4} - \frac{\left(2\omega_0 \cosh[k_0 h]^2 + k_0 c_g\right)^2}{\left(k_0 \sinh[2k_0 h]\right)^2 \left(gh - c_g^2\right)}\right)$$

#### 2.3.2 Wave groupiness

Although extreme events are very important for the extreme loading, there is a difficulty in using them for design as there are not much indication about the probability of occurrence (DNV-GL, 2015). Moreover, not necessarily one single event, although extreme, do represent the most critical situation. Considering the coupling of the wave system with the response of the ship for instance, the passage of a wave group with a sufficient number of waves of critical height and length may be even more dangerous than a single wave, even if of higher amplitude.

An interesting study is under development at Univ. Michigan in which a quite large database of field data has been analyzed with the aim of identifying the wave groups (Seyffert et al. 2016). These real time series could then be the basis for generating an ensemble of wave time series, all of which contained wave groups of known runs and probability of occurrence. The study is not yet mature and consolidated to be used as a practical approach for testing, but it seems a very interesting and promising direction to follow.

### 2.4 Calibration

The wave environment needs to be calibrated prior to the test to ensure the correct environment modelling. The wave effective duration with transient part removed needs to be sufficient as specified. The required tolerances for wave calibration are usually ±5% for both significant wave height and peak wave period.

## 3. GENERATION TECHNIQUES

### 3.1 Regular wave generation

The quality of generated regular waves should be carefully monitored, because their quality degradation may occur quickly during propagation (Benjamin & Feir, 1967; Stansberg, 1993). Also, more attention has to be paid to the passive or active wave absorption of the beach and other boundaries. For the generation of regular waves, second order generation techniques for irregular waves (Schäffer, 1996) can be used. Although second order corrections are very important in the case of steep waves, the second order correction of the flap motions compensate for waves generated due to a mismatch of the flap moton with the wave orbital velocity. This is very relevant for very long and shallow water waves.

Depending upon the application, documentation of possible deviations from ideal conditions such as reflections should be made available from the tests (see and the 22nd ITTC Report on Environmental Modeling). In the wave analysis, stability in time should also be documented, as well as stability in space whenever relevant.

## 3.2 Irregular wave generation

### 3.2.1 Unidirectional wave generation

Unidirectional (or long-crested, 2D) irregular waves are frequently used in most model basins not only because this represents a real seastate in a very simple form, but also because it usually gives a worst case for loadings and responses compared to short-crested (directional) seas. It is also easier to define a sea state in a unique manner.

In the generation of 2D irregular waves, it is important to maintain the randomness that will prevent unrealistic repetition of the waves. Also, careful attention should be given to the effects of the frequency range covered by the servo system. The test duration and the number of frequency components adopted are also important if the proper natural statistics of the wave field are to be reproduced. Wave reflection from the beach and diffraction by the basin wall should be monitored carefully.

Irregular random waves can be modelled as a summation of sinusoidal wave components or as filtering of white noise.

The linear long-crested wave model with the summation of sinusoidal wave components is given by:

$$\eta\ (\mathrm{t}) = \sum_{k=1}^{N} A_k \cos(\varpi_k t + \varepsilon_k)$$

where $\varepsilon_k$ are random phases uniformly distributed between 0 and $2\pi$, mutually independent of each other and of the random amplitudes $A_k$ which can be given by

$$E[A_k^2] = 2S(\varpi_k)\,\Delta\,\varpi_k$$

$S(\varpi_k)$ is the wave spectrum and $\Delta\,\varpi_k = \varpi_k - \varpi_{k-1}$ is the difference between successive frequencies.

No recommended procedure for determining the upper and lower cut-off frequencies has been agreed. One has to minimize the effect of this truncation by carefully selecting the model scale for a given spectrum and wave machine. High frequency truncation lowers the mean period, reduces the bandwidth and is known to affect the slow drift motion due to wave-wave interaction (due to difference frequency effects).

Increasing the number of component frequencies increases the frequency resolution and improves the statistical representation of the waves. The longer the duration of wave generation (determined by the nature of the model tests), the more frequency component are needed. The specified duration of random simulations is important, normally 3 hours for modelling a full storm. This is most often used in offshore engineering tests. It can be changed depending on the phenomena the test is focusing at, however it must be long enough to realize statistical properties if non-linearities and extremes are to be studied. For seakeeping tests, at least 100~200 waves has traditionally been used (typically 0.5~1 hour), which is often defined as satisfactory if linear effects only are considered.

The lowest frequency interval $\Delta\varpi$ can be determined by $\Delta\varpi = \frac{2\pi}{T}$, T is the duration. The number of frequencies to simulate a typical short term sea state should be at least 1000 (DNV, 2010).

### 3.2.2 White noise approach

Another approach to generate irregular sea states is to use a digital white noise *w(t)*, characterized by a density content *W(f)*. By definition of white noise, the power spectral density is $S_w(f) = 1$. An example is provided in Cuong et al. (1982) which is briefly summarized below.

Given the characteristic function of the wave generator, *H(f)*, the problem is to find a function *y(t)* to be used as input to the wave maker in order to obtain the desired spectral density function to be realized, $S_z$. The idea behind the white noise generation approach is that the function *y(t)* can be obtained by *w(t)* through a specific filter *Q(f)*. The filter *Q(f)* may be viewed as the inverse of that needed for whitening the function *y(t)*.

Hence, if Z(f) is the desired frequency content of the wave system to be generated, it is obtained as:

$$Z(f) = H(f) \cdot Q(f) \cdot W(f)$$

and thus the frequency content of *y(t)* is

$$Y(f) = Q(f) \cdot W(f)$$

Correspondingly, the spectral density functions are related by:

$$S_z(f) = |H(f)|^2 |Q(f)|^2 S_w(f)$$

As already stated, $S_w$ can be assumed to be unity and then:

$$S_z(f) = |H(f)|^2 |Q(f)|^2$$

which leads to:

$$|Q(f)|^2 = \frac{S_z(f)}{|H(f)|^2}$$

By introducing the additional constraint that *Q(f)* has to be a real function, the above equation finally provides

$$Q(f) = \frac{\sqrt{S_z(f)}}{|H(f)|}$$

and then:

$$Y(f) = \frac{\sqrt{S_z(f)}}{|H(f)|} W(f)$$

which represents the Fourier transform of the wave maker control time history.

The white noise approach has the advantage of generating non-repeating records.

### 3.2.3 Directional wave generation

Many basins now use multi-directional wave generators to achieve more realistic wave environments. Wave generators usually consist of many small wave boards, which can be controlled independently by electric or electric-hydraulic actuators.

Due to the effects of the Biesel limit on the size of the wave board (Biesel, 1954) and reflection from the wall, wave characteristics in the test region need to be carefully determined. As a reference, the relation of the length of each element of wave makers and the angle of emission of waves is practically shown as follows:

$$l = \frac{\lambda}{\sqrt{2} + \sin\theta}$$

Here, $l$ is the length of each element of wave makers, $\lambda$ is the generated wave length and $\theta$ is the angle of emission of waves.

Modelling directional spectra in the laboratory is generally associated with a significant random scatter, especially in the finer features of the measurements. This reflects features of real sea data, reflecting natural statistical scatter (Stansberg, 1998). Therefore, a robust description of the directional sea conditions is often restricted to a few parameters only, such as mean direction, directional spread, and a simple shape parameter that expresses the bimodality (such as skewness and kurtosis, see Kuik et al., 1988; Stansberg, 2002), or simplified smooth parametric models such as the $cos^{2s}(\theta)$ model.

## 4. REFERENCES

ITTC, (2002) The Special Committee on Waves 2002 Final Report and Recommendations to the 23rd ITTC, In: Proceedings of the 23rd ITTC, pp. 505-736.

ITTC, (1999) The Special Committee on Environmental Modelling Final Report and Recommendations to the 22nd ITTC, In: Proceedings of the 22nd ITTC, pp. 1-38.

ITTC, (2017) The Special Committee on Modelling Environmental Conditions Final Report and Recommendations to the 28th ITTC.

ITTC, (2017) Laboratory modelling of multidirectional, irregular wave spectra, Guideline 7.5-02-07-01.2

Babanin, A.V., (2011) Breaking and Dissipation of Ocean Surface Waves, Cambridge Univ. Press.

Benjamin, T.B., and Feir, J.E., (1967) The disintegration of wave trains on deep water. I Theory, J. Fluid Mechanics, Vol. 27, pp. 417-430.

Biesel, F. (1954) Wave machines, In: Proceedings of the first conference on ships and waves, Hoboken, New Jersey, pp.288-304.

Bretschneider, C.L., (1959) Wave variability and wave spectra for win-generated gravity waves, Beach Erosion Board, U.S. Army Corps of Engineers, Washington (DC), USA. Technical Memorandum No. 118.

Cuong, H.T., Troesch, A.W. and Birdsall, T.G. 1982, "The generation of digital random time series", Ocean Engineering, Vol. 9, pp. 581-588

Chabchoub, A., Hoffmann, N. and Akhmediev, N. (2012) Observation of rogue wave holes in a water wave tank, J. Geophysical Research, Vol. 117, C00J02.

Dean, R.G. and Dalrymple, R.A., (2000) Water Wave Mechanics for Engineers and Scientists, World Scientific.

DNV, (2010) Environmental Conditions and Environmental Loads, Recommended Practice, DNV-RP-C205

DNV, (2011) Modelling and analysis of marine operations, Recommended Practice, DNV-RP-H103.

DNV-GL, (2015) Rogue Waves: impact on ships and offshore structures, DNV-GL Strategic Research and Innovation, Position Paper 05-2015.

Fedele, F., Brennan, J., de León S.P., Dudley, J. and Dias, F., (2016) Real world ocean rogue waves explained without the modulational instability, Scientific Report, Vol. 6, 27715.

Frigaard, P., Helm-Petersen, J., Klopman, G., Stansberg, C.T., Benoit, M., Briggs, M.J., Miles, M., Santas, J., Schäffer. H.A., and Hawkes, P., (1997) IAHR list of sea state parameters – an update for multidirectional waves, In: Proceeding of the IAHR Seminar on Multidirectional Waves and their Interaction with Structures, Seminar proceedings, 27th IAHR Congress.

Henderson, D.M., Patterson, M.S. and Segur, H., (2006) On the laboratory generation of two-dimensional, progressive, surface waves of nearly permanent form on deep water, J. Fluid Mechanics, Vol. 559, pp. 413-427.

Kennedy, A.B. and Fenton, J.D., (1997) A fully-nonlinear computational method for wave propagation over topography, Coastal Engineering, Vol. 32, pp. 137-161.

Kuik, A.J., Van Vledder, G.P., and Holthujsen, L.H., (1988) A method for the routine analysis of pitch-and roll buoy wave data, J. Physical Oceanography, Vol. 18, pp.1020-1034.

Ma, Y., Dong, G., Perlin, M., Ma, X. and Wang, G., (2012) Experimental investigation on the evolution of the modulation instability with dissipation, J. Fluid Mechanics, Vol. 711, pp. 101-121.

Mitsuyasu, H., (1972) The one-dimensional wave spectra at limited fetch, In: Proceedings of the 13th Coastal Engineering Conference, Vol. 1, pp. 289-306.

Newman, J.N., (1977) Marine Hydrodynamics, MIT Press.

Ochi, M.K. and Hubble, E.N., (1976) Six parameter wave spectra, In: Proceedings of the 15th Coastal Engineering Conference, also Coastal Engineering, pp. 301-328.

Onorato, M., Proment, D., Clauss, G. and Klein, M. (2013) Rogue waves: from nonlinear Schrödinger breather solutions to sea-keeping test, PLoS ONE, Vol. 8, e54929.

Rienecker, M.M. and Fenton, J.D., (1981) A Fourier approximation method for steady water waves, J. Fluid Mechanics, Vol. 104, pp. 119-137.

Schäffer, H.A., (1996) Second-Order wave-maker theory for irregular waves, J. Ocean Engineering, Vol. 23, No. 1, 47-88.

Stansberg, C.T., (1993) On spectral instabilities and development of nonlinearities in propagating deep-water wave trains, In: Proceedings of Coastal Engineering 1992, pp. 658-671.

Stansberg, C.T., (1998) Modeling and analysis of multidirectional waves: Statistical variability and directional resolution, In: Proceedings of the Real Sea Conference, KRISO (KR), pp. 22-39.

Stansberg, C.T., (2001) Random waves in the laboratory – what is expected for the extremes, In: Proceeding of the Rogue Waves conference 2000, IFREMER (FR), pp. 289-301.

Tayfun, M. A. and Fedele, F., (2007) Wave-height distributions and nonlinear effects, Ocean Engineering, Vol. 34, pp. 1631-1649.

Toffoli, A., Babanin, A., Bitner-Gregersen, E., Magnusson, A.K., Monbaliu, J., Portilla, J., Dumon, G. and Onorato, M., (2005) On the shape of nonlinear wave trains, In: Proceeding of the Fifth International Symposium on Ocean Waves Measurements and Analysis.

Torsethaugen, K., (1993) A two peak wave spectrum model, In: Proceedings of the 12th International Conference on Ocean, Offshore and Arctic Engineering (OMAE), Vol. II, 175-180.

Tulin, M. P. & Waseda, T., (1999) Laboratory observation of wave group evolution, including breaking effects, J. Fluid Mechanics, Vol. 378, pp. 197-232.