ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Uncertainty Analysis, Example for Open Water Test

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Uncertainty Analysis, Example for Open Water Test

1. PURPOSE OF THE PROCEDURE

The purpose of the procedure is to provide an example for the uncertainty analysis of a model scale towing tank propeller open water test following the ITTC Procedures 7.5-02-01-01, Rev.00, 'Uncertainty Analysis in EFD, Uncertainty Assessment Methodology' and 7.5-02-01-02, Rev.00, 'Uncertainty Analysis in EFD, Guidelines for Towing Tank Tests.'

2. EXAMPLE FOR OPEN WATER TEST

This procedure provides an example showing an uncertainty assessment for a model scale towing tank open water test. A similar example for cavitation tunnel open water tests can also be established. The bias and precision limits and total uncertainties for single and multiple runs have been estimated for the thrust coefficient (K_T) and torque coefficient (K_Q) for one advance coefficient (J). As the speed and rate of revolutions are input parameters to the test, only the bias limit has been established for these quantities providing the bias limit for the corresponding J value.

In order to achieve reliable precision limits, it is recommended that 5 sets of tests with 3 speed measurements in each set are performed giving in total 15 test points. In this example the recommended sequence was followed.

Extrapolation to full scale has not been considered in this example. Although it might lead to significant sources of error and uncertainty, it is not essential for the present purpose of demonstrating the methodology.

When performing an uncertainty analysis for a real case, the details need to be adapted according to the equipment used and procedures followed in each respective facility.

2.1 Test Design

By measuring thrust (T), torque (Q), propeller rate of revolution (n), speed (V) and water temperature ($t°$), the thrust coefficient (K_T), torque coefficient (K_Q) and advance coefficients (J) can be calculated, according to:

$$K_T = \frac{T}{\rho \cdot n^2 D^4} \tag{2-1}$$

$$K_Q = \frac{Q}{\rho \cdot n^2 D^5} \tag{2-2}$$

$$J = \frac{V_A}{nD} \tag{2-3}$$

where ρ is the nominal mass density of the water in the tank set to ρ=1000 kg/m3 according to the ITTC-78 extrapolation method, D is the nominal diameter of the propeller, and V_A is the advance speed of the propeller. T and Q are the measured thrust and torque of the propeller corrected for measured trust and torque of hub and shaft.

2.2 Measurement System and Procedure

Figure 2.1 shows a block diagram for the propeller open water tests including the individual measurement systems, measurement of individual variables, data reduction and experimental results.

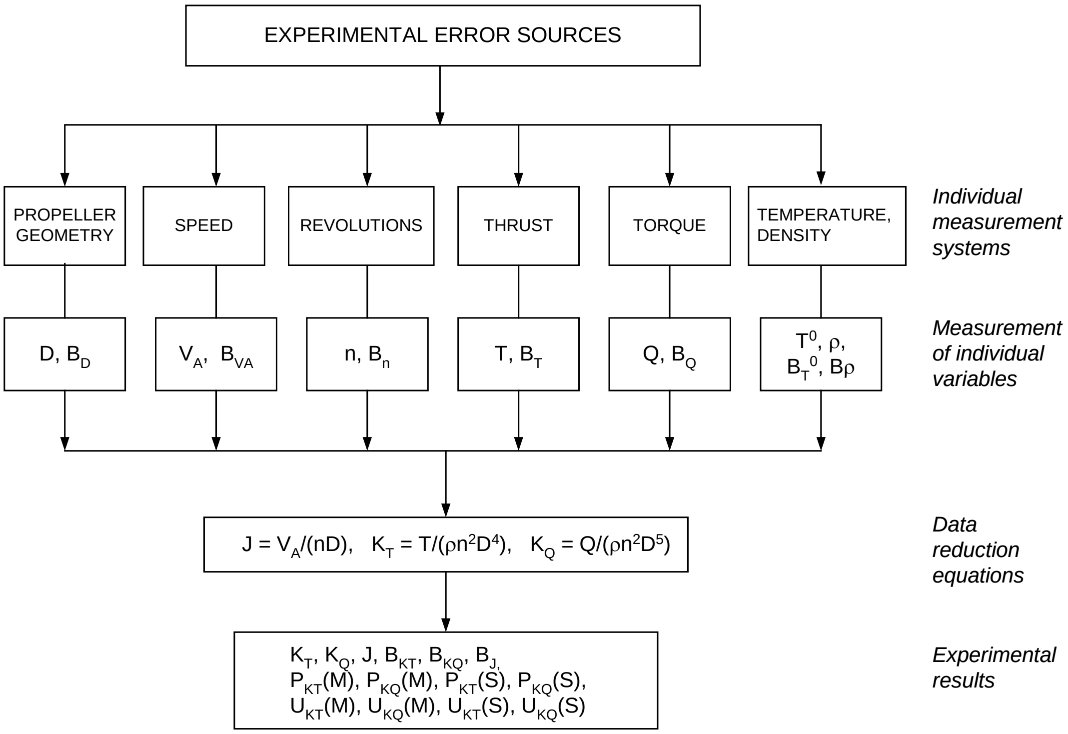

Figure 2.1 Block diagram for the propeller open water tests.

In section 2.3.1 the bias limits contributing to the total uncertainty will be estimated for the individual measurement systems: propeller geometry, speed, thrust, torque, propeller rate of revolution and temperature/density. The elementary bias limits are for each measurement system estimated for the categories: calibration, data acquisition, data reduction and conceptual bias. The bias limits are then, using the data reduction Eq. (2-1), (2-2) and (2-3) reduced into B_{KT}, B_{KQ}, BJ respectively.

The precision limits for the thrust coefficient (P_{KT}) and torque coefficient (P_{KQ}) in model scale are estimated by an end-to-end method for multiple tests (M) and a single run (S).

In Table 2.1 and 2.2 below principal particulars of the model propeller and constants used in the example are tabulated.

Table 2.1 Propeller particulars.

Definitions	Symbol	Value (unit)
No of Prop	*Z*	1 (-)
Propeller diameter	*D*	0.2275 (m)
Propeller Pitch ratio	*P/D*	0.94 (-)
Propeller Blade Area Ratio	*BAR*	0.60 (-)

Table 2.2 Constants.

Definitions	**Symbol**	**Value (unit)**
Gravity	*g*	9.81 (m/s^2)
Density, model basin	ρ	1000 (kg/m^3)
Water temperature (POW test average)	t^{o}	15.0 (degrees)

2.3 Uncertainty Analysis

The total uncertainty for the thrust and torque coefficients are given by the root sum square of the uncertainties of the total bias and precision limits.

$$\left(U_{K_T}\right)^2 = \left(B_{K_T}\right)^2 + \left(P_{K_T}\right)^2 \tag{2-4}$$

$$\left(U_{K_Q}\right)^2 = \left(B_{K_Q}\right)^2 + \left(P_{K_Q}\right)^2 \tag{2-5}$$

The bias limit for Eq. (2-1) (2-2) and (2-3) is

$$\left(B_{K_T}\right)^2 = \left[\frac{\partial K_T}{\partial D} B_D\right]^2 + \left[\frac{\partial K_T}{\partial n} B_n\right]^2 + \left[\frac{\partial K_T}{\partial T} B_T\right]^2 + \left[\frac{\partial K_T}{\partial \rho} B\rho\right]^2 \tag{2-6}$$

$$\left(B_{K_Q}\right)^2 = \left[\frac{\partial K_Q}{\partial D} B_D\right]^2 + \left[\frac{\partial K_Q}{\partial n} B_n\right]^2 + \left[\frac{\partial K_Q}{\partial Q} B_Q\right]^2 + \left[\frac{\partial K_Q}{\partial \rho} B\rho\right]^2 \tag{2-7}$$

$$\left(B_J\right)^2 = \left[\frac{\partial J}{\partial V_A} B_{V_A}\right]^2 + \left[\frac{\partial J}{\partial n} B_n\right]^2 + \left[\frac{\partial J}{\partial D} B_D\right]^2 \tag{2-8}$$

The precision limits will be determined for K_T and K_Q by an end-to-end method where all the precision errors for speed, thrust, torque, revolution of propeller and temperature/density are included. The precision limits for a single run (S) and for the mean value of multiple tests (M) are determined. Regardless as to whether the precision limit is to be determined for single or multiple runs the standard deviation must be determined from multiple tests in order to include random errors. If it is not possible to perform repeated tests the experimenter must estimate a value for the precision error using the best information available at that time.

The precision limit for multiple tests is calculated according to

$$P(M) = \frac{K\ SDev}{\sqrt{M}} \tag{2-9}$$

where *M*=number of runs for which the precision limit is to be established, S_{Dev} is the standard deviation established by multiple runs and *K*=2 according to the methodology.

The precision limit for a single run can be calculated according to

$$P(S) = K \cdot S_{\text{Dev}} \tag{2-10}$$

2.3.1 Bias Limit

Under each group of bias errors (propeller geometry, speed of advance, propeller rate of revolution, thrust, torque, temperature/density) the elementary error sources have been divided into the following categories: calibration; data acquisition, data reduction; and conceptual bias. The categories not applicable for each respective section have been left out.

2.3.1.1 Propeller Geometry

The model is manufactured to be geometrically similar to the actual propeller geometry. Although efforts are made to produce an accurate propeller model, including the use of NC milling machines, errors in dimensions and offsets can occur leading, for example, to errors in diameter, chord length, pitch and blade section shape. The influence of these errors in dimensions and shape can strongly affect the flow characteristics around the propeller blades and hence the measured thrust and torque. These errors can only be estimated through systematic variations in propeller geometry and offsets. For example, the sensitivity of pitch setting can be estimated by performing multiple open water tests, where the pitch has been re-set between each set of tests. Errors arising from blade shape inaccuracies can only be estimated by performing multiple tests with different propeller models manufactured from the same surface description.

It is seen that many of the errors in propeller geometry are difficult to estimate and are therefore not considered in this example. Only the bias error on propeller diameter will be considered.

Data acquisition:

In this example the error in model propeller diameter due to manufacturing error is estimated from the limits given in ITTC Procedure 7.5-01-01-01 Rev 01, ‘Ship Models.’ By assuming the error in model diameter to be within ±0.1mm the bias error is estimated to be BD=0.1 mm corresponding to 0.044% of the nominal diameter of 227.5 mm. The bias error in propeller diameter does in this example therefore only affect the data reduction equations for thrust and torque coefficients.

2.3.1.2 Speed of Advance of the Propeller

The carriage speed measurement system consists of individual measurement systems for pulse count (c), wheel diameter (D) and 12 bit DA and AD card time base (Δt). The speed is determined by tracking the rotations of one of the wheels with an optical encoder. The encoder is perforated around its circumference with 8000 equally spaced and sized windows. As the wheel rotates, the windows are counted with a pulse counter. The speed circuit has a 100 ms time base, which enables an update of the pulse every 10^{th} of a second. A 12-bit DA conversion in the pulse count limits the maximum number of pulses in 100 ms to 4096. The output of the speed circuit is 0-10 V so that 4096 counted in 100 ms corresponds to 10 V output. The output from the encoder is calculated with the equation

$$V = \frac{c\pi D}{8000\Delta t} \tag{2-11}$$

where c is the number of counted pulses in Δt=100 ms and D is the diameter of the carriage wheel (0.381 m).

The bias limit from blockage effects has not been considered.

<u>Pulse count (*c*)</u>

Calibration:

The optical encoder is factory calibrated with a rated accuracy of ±1 pulse on every update. This value is a bias limit and represents the minimum resolution of the 12-bit AD data acquisition card. Therefore, the bias limit associated with the calibration error will be B_{c1}=1 pulse (10V/212=0.00244 V).

Data acquisition:

In the given data acquisition cycle, the speed data is converted to the PC by two 12-bit conversions. The resolution is *resol*=10 V/ 212 = 0.00244V / bit. The AD boards are accurate to 1.5 bits or pulses, which was determined by calibrating the boards against a precision voltage source. Therefore, the bias associated with the two conversions is B_{c2}= B_{c3}=1.5 pulses (0.00366 V).

Data reduction:

The final bias occurs when converting the analogue voltage to a frequency that represents the pulse count over 10 time bases or one second. This is enabled if correlating the given frequency to a corresponding voltage output. The bias limit results from approximating a calibration (set of data) with a linear regression curve fit. The statistic is called standard error estimate (SEE) and is written from Coleman and Steele (1999) as

$$SEE = \sqrt{\frac{\sum_{i=1}^{N}\left(Y_i - (aX_i + b)\right)^2}{N-2}} \tag{2-12}$$

It is proposed by Coleman and Steele (1999) that *a* ±2(SEE) band about the regression curve will contain approximately 95% of the data points and this band is a confidence interval on the curve fit. The curve fit bias limit is calculated to be 2.5 Hz corresponding to B_{c4}= 0.25 pulse (0.000614 V).

The total bias limit for pulse count will then be

$$\begin{aligned} B_c &= \left(B_{c1}^2 + B_{c2}^2 + B_{c3}^2 + B_{c4}^2\right)^{\frac{1}{2}} = \\ &\left(1^2 + 1.5^2 + 1.5^2 + 0.25^2\right)^{\frac{1}{2}} = \\ &2.358\,\text{pulse}\,(0.00576\,\text{V}) \end{aligned} \tag{2-13}$$

<u>Wheel diameter (*D*)</u>

One of the driving wheels of the carriage is used for the speed measurement. The wheel is measured with constant time intervals to ensure the right calibration constant is used.

Calibration:

The wheel diameter is measured with a high quality Vernier calliper at three locations at the periphery of the wheel which are averaged for a final value of *D*. The wheel diameter is considered accurate to within B_D=0.000115 m.

Time base (Δt)

The time base of the speed circuitry is related to the clock speed of its oscillator module.

Calibration:

The oscillator module is factory calibrated and its rated accuracy is 1.025 10-5 seconds on every update giving $B_{\Delta t}$= 1.025 10-5 seconds.

The data reduction equation is derived from Eq. (2-11) and can be written

$$B_V=\left(\left(\frac{\partial V}{\partial c}B_c\right)^2+\left(\frac{\partial V}{\partial D}B_D\right)^2+\left(\frac{\partial V}{\partial \Delta t}B_{\Delta t}\right)^2\right)^{\frac{1}{2}} \quad (2\text{-}14)$$

Using the nominal values of *c*=1283, *D*=0.381 m and Δt=0.1 s for the mean speed of *V*=1.9197 m/s the partial derivatives can be calculated as

$$\frac{\partial V}{\partial c}=\frac{\pi D}{8000\Delta t}=0.00150 \quad (2\text{-}15)$$

$$\frac{\partial V}{\partial D}=\frac{c\pi}{8000\Delta t}=5.0385 \quad (2\text{-}16)$$

$$\frac{\partial V}{\partial \Delta t}=\frac{c\pi D}{8000}\left(-\frac{1}{\Delta t^2}\right)=-19.1967 \quad (2\text{-}17)$$

The total bias limit can then be calculated as

$$B_V=\left(\begin{array}{l}(0.00150\cdot 2.358)^2+(5.0385\cdot 0.000115)^2\\+(19.1967\cdot 1.025\mathrm{E}^{-5})^2\end{array}\right)^{\frac{1}{2}}$$

$$=0.00358 \quad (2\text{-}18)$$

The total bias limit for the speed is B_V=0.00358 m/s corresponding to 0.19% of the nominal speed of 1.9197m/s.

The bias limit for the speed could alternatively be determined end-to-end, by calibrating against a known distance and a measured transit time.

2.3.1.3 Propeller Rate of Revolution

The propeller rate of revolution measurement system consists of individual measurement systems for pulse count (*c*) and 12 bit AD conversion at a time base (Δt). The rate of rotation is sensed by tracking the rotation of propeller shaft with an optical encoder. The encoder has a pulse capacity of 600 due to equally spaced and sized windows at one revolution. As the wheel rotates, the windows are counted with the encoder. The speed circuit has a 100 ms time base which enables updates of the signal every 10th of a second. A 12-bit DA conversion in the pulse count limits the maximum number of pulses in 100 ms to 4096. The output of the speed circuit is 0-10 V so that 4096 counted in 100 ms corresponds to 10 V output. The output from the encoder is calculated with the equation

$$n = \frac{c}{No_{\text{Pulse}} * \Delta t} \tag{2-19}$$

where c is the number of counted pulses in Δt=100 ms and NoPulse is the number of pulses at one revolution (600).

Pulse count (c)

Calibration:

The optical encoder is factory calibrated with a rated accuracy of ±1 pulse on every update. This value is a bias limit and represents the minimum resolution of the 12-bit AD data acquisition card. Therefore, the bias limit associated with the calibration error will be B_{C1}=1 pulse (10V/212=0.00244 V).

Data acquisition:

In the given data acquisition cycle, the rate of revolution pulse data is converted to DC signal and acquired by the PC through one 12-bit conversion. The resolution is *resol*=10 V/ 212 = 0.00244V / bit. The conversion is accurate to 1.5 pulses and AD boards are accurate to 1.5 bits or pulses, which was determined by calibrating the boards against a precision voltage source. Therefore, the bias associated with the conversions is B_{C2}=B_{C3}= 1.5 pulses (0.00366 V).

The total bias limit for pulse count will then be

$$B_C = \left(B_{C1}^2 + B_{C2}^2 + B_{C3}^2\right)^{\frac{1}{2}} = \left(1^2 + 1.5^2 + 1.5^2\right)^{\frac{1}{2}} = 2.345\,\text{pulse} \tag{2-20}$$

Time base (Δt)

The time base of the speed circuitry is related to the clock speed of its oscillator module.

Calibration:

The oscillator module is factory calibrated and its rated accuracy is 1.025 10-5 seconds on every update giving $B_{\Delta t}$= 1.025 10-5 seconds.

The data reduction equation is derived from Eq. (2-19) and can be written

$$B_n = \left(\left(\frac{\partial n}{\partial c} B_c \right)^2 + \left(\frac{\partial n}{\partial \Delta t} B_{\Delta t} \right)^2 \right)^{\frac{1}{2}} \tag{2-21}$$

Using the nominal values of c=840, and Δt=0.1 s for the mean speed of n=14.00 rps the partial derivatives can be calculated as

$$\frac{\partial n}{\partial c} = \frac{1}{600\Delta t} = 0.01667 \tag{2-22}$$

$$\frac{\partial n}{\partial \Delta t} = \frac{c}{600}\left(-\frac{1}{\Delta t^2}\right) = -140 \tag{2-23}$$

The total bias limit can then be calculated from Eq. (2-21) as

$$B_n = \left((0.01667 \cdot 2.345)^2 + \left(140 \cdot 1.025\,10^{-5}\right)^2 \right)^{\frac{1}{2}} = 0.0391 \tag{2-24}$$

The total bias limit associated with the propeller rate of revolution is B_n=0.0391 rps corresponding to % 0.279 of the nominal rate of revolution of 14.00 rps.

2.3.1.4 Torque and Thrust

The axial force (thrust) and the moment around propeller axis (torque) are to be measured when the model propeller rotates and is towed through the water.

Calibration:

The thrust transducer is calibrated with weights. The weights are the standard for the load cell calibration and are a source of error, which depends on the quality of the standard. The weights have a certificate that certifies their calibration to a certain class. The tolerance for the individual weights used is certified to be ± 0.005%.

The bias error arising from the tolerance of the calibration weights, B_{Tw}, is calculated as the accuracy of the weights, times the measured thrust according to Eq. (2-25).

$$B_{Tw} = 0.00005 \cdot 100.4218 = 0.0050\text{ N} \quad (2\text{-}25)$$

The torque dynamometer is calibrated with weights applied at a nominal distance from the shaft line. The weights have the same tolerance as for the thrust calibration i.e. ± 0.005%. The moment arm is 0.125 m with accuracy of ± 0.2 mm.

As moment is calculated by the equation

$$Q_w = F\ ma \quad (2\text{-}26)$$

the bias error can be written as

$$\left(B_{Qw}\right)^2 = \left(\frac{\partial Q_w}{\partial F} B_F\right)^2 + \left(\frac{\partial Q_w}{\partial ma} B_{ma}\right)^2 \quad (2\text{-}27)$$

or

$$\left(B_{Qw}\right)^2 = \left(ma \cdot B_F\right)^2 + \left(F \cdot B_{ma}\right)^2 \quad (2\text{-}28)$$

If F=*Q*/*ma*=3.6747 (Nm) /0.125 (m) = 29.398 N, B_F = 0.00005·29.398=0.001470 N, *ma*=0.125 m and B_{ma}=0.0002 m the data reduction can be written as:

$$B_{Qw} = \sqrt{\left(0.125 \cdot 0.001470\right)^2 + \left(29.398 \cdot 0.0002\right)^2} = 0.0059 \quad (2\text{-}29)$$

The bias limit arising from the inaccuracy in calibration torque would therefore be B_{Qw} =0.0059 Nm.

Data acquisition:

The resolution of the 12 bit AD converter is 1 pulse. Using the slope of the respective calibration curves the bias limits can be calculated to:

$$B_{T_{AD}} = \frac{1 \cdot 10}{4096} 78.247(\text{slope}) = 0.191\text{ N} \quad (2\text{-}30)$$

$$B_{Q_{AD}} = \frac{1 \cdot 10}{4096} 2.936(\text{slope}) = 0.0072\text{ Nm} \quad (2\text{-}31)$$

The data from the calibration in Tables 2.3 and 2.4 show the mass/volt and moment/volt relations. From these values the 2**SEE* can be calculated with Eq. (2-12) to B_{Tcf}=0.3149 N for thrust and B_{Qcf}=0.0098 Nm for torque.

Table 2.3 Thrust transducer calibration.

Thrust (kg)	Mass (kg)	Output (Volt)
0	0	0.000
10	10	1.255
20	20	2.509
30	30	3.763
40	40	5.017
50	50	6.272
0	0	0.006
-10	-10	-1.248
-20	-20	-2.503
-30	-30	-3.758
-40	-40	-5.013
-50	-50	-6.268

T(N)= -0.2087+Volt·78.247

Table 2.4 Torque transducer calibration.

Torque (kgm)	Mass (kg)	Output (Volt)
0.0	0	0.000
0.5	4	1.668
1.0	8	3.340
1.5	12	5.012
2.0	16	6.685
0.0	0	0.000
-0.5	-4	-1.669
-1.0	-8	-3.339
-1.5	-12	-5.010
-2.0	-16	-6.682

Q(Nm)= -0.0015+Volt·2.936

The bias limit for thrust and torque become

$$B_T = \sqrt{((B_{Tw})^2 + (B_{T_{AD}})^2 + (B_{Tcf})^2)} = \sqrt{(0.0050)^2 + (0.1910)^2 + (0.3149)^2} = 0.3683\,\text{N}$$

(2-32)

$$B_Q = \sqrt{((B_{Qw})^2 + (B_{Q_{AD}})^2 + (B_{Q_{cf}})^2)} = \sqrt{(0.0059)^2 + (0.0072)^2 + (0.0098)^2} = 0.0135\,\text{Nm}$$

(2-33)

2.3.1.5 Temperature/Density

Temperature

Calibration:

The thermometer is calibrated by the manufacturer with a guaranteed accuracy of ±0.30 °C within the interval -5 to +50 °C. The bias error limit associated with temperature measurement is B_{t^o} =0.°C corresponding to 2 % of the nominal temperature of 15 °C.

Density

Calibration:

The density-temperature relationship (table) according to the ITTC Procedure 7.5-02-01-03 Rev 00 ‘Density and Viscosity of Water’ for g=9.81m/s² can be expressed as:

$$\rho = 1000.1 + 0.0552 \cdot t^o - 0.0077 \cdot t^{o2} + 0.00004 t^{o3}$$

(2-34)

$$\left|\frac{\partial \rho}{\partial t}\right| = \left|0.0552 - 0.0154 t^\circ + 0.000120 t^{\circ 2}\right| \quad (2\text{-}35)$$

Using Eq. (2-35) with t^o=15 °C and Bt° = 0.3 °C the bias $B_{\rho 1}$ can be calculated according to:

$$B_{\rho 1} = \left|\frac{\partial \rho}{\partial t^{\circ}}\right| B_{t^{\circ}} = 0.1488 \cdot 0.3 = 0.04464\,\text{kg/m}^3 \quad (2\text{-}36)$$

Data reduction:

The error introduced when converting the temperature to a density (table lookup) can be calculated as two times the *SEE* of the curve fit to the density/temperature values for the whole temperature range. Comparing the tabulated values with the calculated values (Eq. 2-34) the bias error $B_{\rho 2}$ can be calculated as $B_{\rho 2}$=0.070 kg/m^3.

Conceptual:

The nominal density according to the ITTC-78 method is ρ = 1000kg/m³. Using this method introduces a bias limit as the difference between ρ (15 °C) = 999.34 kg/m³ and ρ = 1000 kg/m³ such that $B_{\rho 3}$ = 1000.0-999.345 = 0.655 kg/m^3 corresponding to 0.0655% of the density.

The bias for ρ can then be calculated according to:

$$B_\rho = \sqrt{\left(B_{\rho 1}\right)^2 + \left(B_{\rho 2}\right)^2 + \left(B_{\rho 3}\right)^2} = \sqrt{(0.01488 \cdot 0.3)^2 + 0.070^2 + 0.655^2} = 0.660\,\text{kg/m}^3 \quad (2\text{-}37)$$

The bias limit for density is thus B_ρ=0.660 kg/m^3 corresponding to 0.066 % of ρ = 1000 kg/m³. If using the density value determined by the temperature, the bias limit $B_{\rho 3}$ will be eliminated.

2.3.1.6 Total Bias Limits

In order to calculate the total bias and precision limits, partial derivatives have to be calculated using input values of T = 100.422 N, Q = 3.6747 Nm, n = 14.00 rps, V = 1.9197 m/s, D=0.2275 m and ρ=1000 kg/m³.

For this condition, partial derivatives for KT are,

$$\frac{\partial K_T}{\partial D} = \frac{T}{\rho \cdot n^2}\left(-\frac{4}{D^5}\right) = -3.3607 \quad (2\text{-}38)$$

$$\frac{\partial K_T}{\partial n} = \frac{T}{\rho \cdot D^4}\left(-\frac{2}{n^3}\right) = -0.02731 \quad (2\text{-}39)$$

$$\frac{\partial K_T}{\partial T} = \frac{1}{\rho \cdot n^2 D^4} = 0.001904 \quad (2\text{-}40)$$

$$\frac{\partial K_T}{\partial \rho} = \frac{T}{n^2 D^4}\left(-\frac{1}{\rho^2}\right) = -0.000191 \quad (2\text{-}41)$$

Partial derivatives for K_Q are,

$$\frac{\partial K_Q}{\partial D} = \frac{Q}{\rho \cdot n^2}\left(-\frac{5}{D^6}\right) = -0.6756 \quad (2\text{-}42)$$

$$\frac{\partial K_Q}{\partial n} = \frac{Q}{\rho \cdot D^5}\left(-\frac{2}{n^3}\right) = -0.004392 \quad (2\text{-}43)$$

$$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho \cdot n^2 D^5} = 0.008366 \quad (2\text{-}44)$$

$$\frac{\partial K_Q}{\partial \rho} = \frac{Q}{n^2 D^5}\left(-\frac{1}{\rho^2}\right) = -0.000031 \qquad (2\text{-}45)$$

Partial derivatives for *J* are,

$$\frac{\partial J}{\partial V} = \frac{1}{nD} = 0.3139 \qquad (2\text{-}46)$$

$$\frac{\partial J}{\partial n} = \frac{V}{D}\left(-\frac{1}{n^2}\right) = -0.0430 \qquad (2\text{-}47)$$

$$\frac{\partial J}{\partial D} = \frac{V}{n}\left(-\frac{1}{D^2}\right) = -2.6486 \qquad (2\text{-}48)$$

The total bias limit can then be calculated according to Eq. (2-6), (2-7) and (2-8) as

$$B_{K_T} = 0.001327 \qquad (2\text{-}49)$$

$$B_{K_Q} = 0.0002175 \qquad (2\text{-}50)$$

$$B_J = 0.00204 \qquad (2\text{-}51)$$

corresponding to 0.694 % of the thrust coefficient K_T=0.1912 , 0.707 % of the torque coefficient K_Q=0.03074, and 0.339% of the advance coefficient J=0.6026.

2.3.2 Precision Limit

In order to establish the precision limits, the standard deviation for a number of tests, with the model removed and reinstalled between each set of measurements, must be determined. In this example, 5 sets of testing (A-E) with 3 speed measurements in each set have been performed giving in total 15 test points. The measurements have been performed for 3 advance coefficients.

Performing repeat tests with multiple set-ups are a suitable way to include random errors in the set-up. The measurement has been performed for two Reynolds numbers to take possible scale effects into account.

When the measurements are repeated it is also likely that the measured quantities will be taken at slightly different speeds and rpm for the different runs, and with different water temperatures between the different set of tests.

Table 2.5 Precision limits of thrust and torque coefficients for all data

Series /run	**Thrust and Torque coefficient based on measured values corrected to *J*=0.55**		**Thrust and Torque coefficient based on measured values corrected to *J*=0.60**		**Thrust and Torque coefficient based on measured values corrected to *J*=0.65**	
	K_T	K_Q	K_T	K_Q	K_T	K_Q
A1	0.21608	0.03308	0.19220	0.03018	0.16888	0.02735
A2	0.21615	0.03309	0.19259	0.03026	0.16887	0.02736
A3*	0.21615	0.03296	0.19239	0.03009	0.16894	0.02718
B1	0.21658	0.03311	0.19265	0.03024	0.16875	0.02730
B2	0.21651	0.03315	0.19210	0.03017	0.16853	0.02729
B3*	0.21634	0.03298	0.19224	0.03007	0.16893	0.02719
C1	0.21617	0.03311	0.19233	0.03019	0.16890	0.02736
C2	0.21610	0.03307	0.19254	0.03024	0.16862	0.02730
C3*	0.21653	0.03299	0.19263	0.03010	0.16888	0.02719
D1	0.21623	0.03307	0.19239	0.03019	0.16864	0.02735
D2	0.21630	0.03308	0.19233	0.03016	0.16861	0.02729
D3*	0.21632	0.03295	0.19216	0.03006	0.16858	0.02722
E1	0.21660	0.03315	0.19278	0.03024	0.16852	0.02729
E2	0.21610	0.03308	0.19265	0.03024	0.16854	0.02730
E3*	0.21619	0.03294	0.19218	0.03006	0.16897	0.02718
MEAN	0.21629	0.03305	0.19241	0.03017	0.16874	0.02728
SDev	0.00018	0.00007	0.00021	0.00007	0.00017	0.00007
P(S) (%)	0.00037 (0.17035)	0.00014 0.43173	0.00043 0.22150	0.00015 0.48317	0.00034 0.20324	0.00014 0.49654
P(M) (%)	0.00010 (0.04398)	0.00004 0.11147	0.00011 0.05719	0.00004 0.12475	0.00011 0.06775	0.00003 0.12821

* A3,B3,C3,D3 and E3 are the data for the high Reynolds Number Re=0.9·10^6.

The differences in speed and rpm will occur due to the difficulties in setting the exactly correct speed and rpm. In order to compare the data, the thrust and torque coefficients can be calculated for each measured point and thereafter corrected to the correct J value by using the slope of the K_T and K_Q curve. The deviations in speed and rpm should not be interpreted as precision errors as they are input parameters. The K_T and

K_Q measurements are simply taken at different J values i.e. different conditions.

A correction for difference in temperature between the different sets of tests is not carried out. Such a correction would incorrectly change the uncertainty, as the temperature of the open water test is not considered in the extrapolation to full scale when using the ITTC-78 method.

In Table 2.5, K_T and K_Q have been calculated based on measured quantities for the three J values J=0.55, 0.60 and 0.65 for two Reynolds numbers (Re=0.5·106 and Re=0.9·106).

In the Table 2.5 the mean values, standard deviation (S_{Dev}), precision limits for single run Eq. (2-10), and precision limits for multiple runs Eq. (2-9) have been calculated. The corresponding percentage values have also been given within brackets. Note that the precision limits used in the following calculations are based on the standard deviations calculated using results from two Reynolds numbers.

2.4 Total Uncertainties

The precision limits of the propeller open-water characteristics curves might be selected at a certain Reynolds number or certain Reynolds number range. The decision depends on the propulsion tests and full-scale extrapolation procedures. In this case the precision limits based on both Reynolds numbers have been used.

Combining the precision limits for multiple and single tests with the bias limits the total uncertainty can be calculated according to Eq. (2-4) and Eq. (2-5) for J = 0.60.

The total uncertainty for the mean value of 15 tests will be calculated as

$$U_{\overline{K_T}} = \sqrt{(0.001327)^2 + (0.000110)^2} = 0.001332 \tag{2-52}$$

$$U_{\overline{K_Q}} = \sqrt{(0.0002175)^2 + (0.00003764)^2} = 0.0002207 \tag{2-53}$$

which is 0.70 % of K_T and 0.72 % of K_Q.

Correspondingly, the total uncertainty for a single run will be calculated as:

$$U_{K_T} = \sqrt{(0.001327)^2 + (0.000426)^2} = 0.001394 \tag{2-54}$$

$$U_{K_Q} = \sqrt{(0.0002175)^2 + (0.0001458)^2} = 0.0002618 \tag{2-55}$$

which is 0.73 % of K_T and 0.85 % of K_Q.

Expressed as relative numbers the bias for KT represents 90.6 % of the total uncertainty for a single run and 99.3 % of the total uncertainty for the mean value of 15 tests. The bias for K_Q represents 69.0% of the total uncertainty for a single run and 97.1 % of the total uncertainty for the mean value of 15 tests.

By comparing the bias and precision limits and the uncertainties, the relative contribution of each term can be calculated. This makes it possible to determine where an upgrade in the measurement system has the largest effect. Compare ITTC Procedure 7.5-02-02-02 Rev 01, 'Uncertainty Analysis, Example for Resistance Test.'

3. REFERENCES

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