ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Determination of a type A uncertainty estimate of a mean value from a single time series measurement

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-01	General
7.5-02-01-06	Determination of a type A uncertainty estimate of a mean value from a single time series measurement

Updated / Edited by	Approved
Quality Systems Group	28th ITTC 2017
Date: 07/2016	Date: 09/2017

Table of Contents

Estimation of uncertainty derived from a single time series measurement

1. PURPOSE OF PROCEEDURE

Single time histories such as those obtained from measurements in a towing tank, wind tunnel or full scale ship trail are often subjected to low frequency random disturbances caused by long transient responses to startup conditions (e.g. towing carriage acceleration profile), low damping, recirculation effects and a host of external factors such as varying environmental conditions. Such disturbances cause a random low frequency variation in the mean value of the measured time history.

Accurate estimation of the mean value can be problematic in cases where the low frequency random variation in the mean value is large compared to the estimated mean value. The 'traditional' approach to obtaining a 'good' estimate for the mean value of such 'non-stationary' signals is to increase the length of the measurement time, requiring a very long towing tank/test region. The assumption with this approach being that the non-stationary behavior diminishes with time, which may not always be the case. The more common practice is to carry out multiple repeat tests and combine the results in order to obtain a mean value within acceptable limits. However the requirement to carry out multiple repeat tests is often at odds with the commercial realities of limited time and test budgets.

Methods have been suggested in Lin et al. (1990) and in Molloy (2010) which process a single time series and obtain an estimate the level of uncertainty. These and other traditional methods require subjective choices, such as the number and length of the subdivision of the time series, to be made. As such these methods are difficult to proceduralise.

More recently Brouwer et al (2013), (2015a) proposed two objective procedures for estimating the uncertainty from a single time series measurement based on auto covariance and segment method, the former being favored by the authors for its rapid convergence. Brouwer, Tukker and van Rijsbergen proposed their 'Transient Scanning Technique', Brouwer et al (2015b). This technique allows to determine whether a signal approaches a stationary state or not and also provides information to allow non-stationary behaviour (typically from start up and end effects) to be removed from the signal so that the uncertainty in the mean value can be estimated using the methods in Brouwer et al (2015a) more accurately. A summary of this work comprises the majority of this document.

The TST is a practical and simple tool to verify whether the mean value is constant or not. Further, it locates trends (or transients) in measured signals at the beginning or at the end of time series. These transients are often too small to see in visual checks, but with the TST these transients can be identified and hence removed from the signal. The residual part of the time series is stationary and can be used for further analysis.

Section 2 explains briefly the theoretical background. In Section 3, the TST is described. In Section 4 the technique demonstrated using synthetic data. Section 5 provides a step-by-step guide to implementing the procedure. References are provided in section 6.

2. DESCRIPTION OF PROCEDURE

2.1 Theoretical background

This section provides a brief description of the theoretical background presented in detail in Brouwer et al (2013) and Brouwer et al (2015b).

A time series $x_i(t)$ with finite length T is considered as a sample record of an ergodic stationary random process. The sample average m_i is an estimate of the true mean of the process μ_x.

$$m_i = \frac{1}{T}\int_0^T x_i(t)dt \qquad 1$$

The sample variance s_i^2 is an estimate of the true variance of the process σ_x^2

$$s_i^2 = \frac{1}{T}\int_0^T (x_i(t) - m_i)^2 dt \qquad 2$$

Due to the finite length T of the time series there will be a difference between the estimated mean m_i and the true mean μ_x.

The expected value of the variance of the mean can be written as

$$s_m^2 = E[(m_i - \mu_x)^2] \qquad 3$$

Substituting equation 1 into equation 3 and after some manipulation following Bendat and Piersol (2010), the standard deviation of the mean can be written as:

$$s_m = \sqrt{\frac{2}{T}\int_0^T \left(1 - \frac{\tau}{t}\right) C_{xx}(\tau)d\tau} \qquad 4$$

where:

s_m is the standard deviation of the mean, (a measure of the standard uncertainty of mean: u_1 = s_m)

T is the measurement length

$C_{xx}(\tau)$ is the autocovariance function for a stationary process

τ is the time difference or lag

In Brouwer et al (2013) it was shown how to compute the expected s_m, for ***stationary*** stochastic processes with arbitrary spectral shapes when T is sufficiently long. It also showed that for such processes, s_m decays as the sample length T increases.

3. TRANSIENT SCANNING TECHNIQUE

In Brouwer et al (2013) two methods were presented to estimate s_m from a single signal, the autocovariance and segment method. Both methods provide an estimate for s_m called u_1, the standard uncertainty of the mean. The inverse relation between u_1 and T can be verified visually in a graph with T plotted on the x-axis and u_1 on the y-axis, both with logarithmic scales. For stationary signals, the trend should form a line with a slope of minus one. If the slope differs from minus one then the signal is non- stationary.

The autocovariance method was found to have better convergence properties as well as giving less scatter than the segment method. The autocovariance method uses equation 4 in a modified form

$$u_1 = \sqrt{\frac{1}{T}\int_0^T \left(1 - \frac{\tau}{t}\right) C_{xx,biased}(\tau)d\tau} \qquad 5$$

Where $C_{xx,biased}(\tau)$ is the biased estimator for the autocovariance - used to reduce numerical instability for large values of τ, Brouwer et al (2013).

$$C_{xx,biased}(\tau) = \left(1 - \frac{|\tau|}{T}\right) . C_{xx}(\tau) \qquad 6$$

In Brouwer et al (2013) the auto covariance method was used to develop a technique, called the Transient Scanning Technique, TST. The TST identifies regions in the time history which exhibit non- stationary behavior and allows these regions to be removed from the signal analysis on order to obtain a more accurate estimate of the mean value. A TST is constructed by calculating the cumulative u_1 as shown in equation 5 for a range of sample lengths T. The TST can be applied in two ways.

3.1 TST-A

Starting from the beginning of the signal t_{begin} the TST selects signal sections [t_{begin}, $t_{begin} + T$] for each section size T and calculates the cumulative u_1 using equation 5. The results are plotted against T on a logarithmic scale. TST-A is very strong in identifying end effects in the signal.

3.2 TST-B

Starting from the end of the signal $t = t_{tend}$ the TST selects signal sections [t_{end} – T, t_{end}] for each T and calculates the cumulative u_1 using equation 5. The results are plotted against T. TST-B is very strong in identifying start-up effects in the signal.

4. EXAMPLE

To demonstrate the TST, a realisation is picked from an artificially created stochastic process. Its mean is zero and it has no energy below 0.25 Hz. The realisation length is 100 s, which is long enough to be stationary. A second signal is created from the realisation, which is made non-stationary by adding a small start-up effect in the first 20 s. Both signals and the disturbance are shown in Figures 4.1, 4.2 and 4.3. More details can be found in Brouwer et al (2015b).

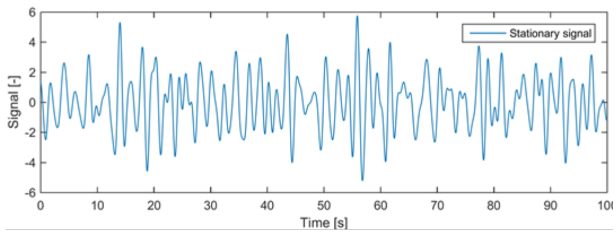

Figure 4.1 Realisation taken from a stochastic process.

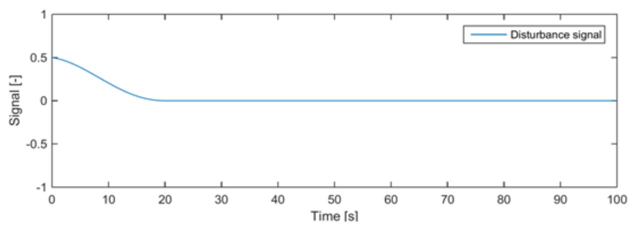

Figure 4.2 Start-up disturbance.

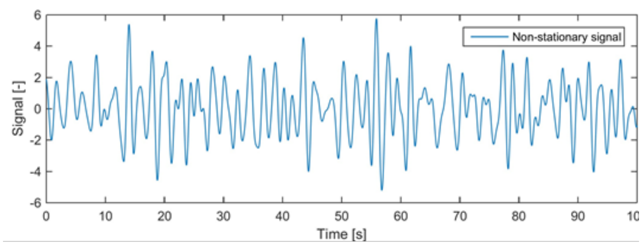

Figure 4.3 Realisation with added start-up disturbance.

Checking for stationarity by visual inspection of the signal in Figure 4.3 is difficult since the start-up effect has much smaller amplitude than the signal. Figure 4.4 shows the TST results for the stationary signal and the TST for the non-stationary signal is shown in Figure 4.5. In order to identify the startup effects the cumulative u1 values are calculated from the end (TST-B), using the autocovariance method.

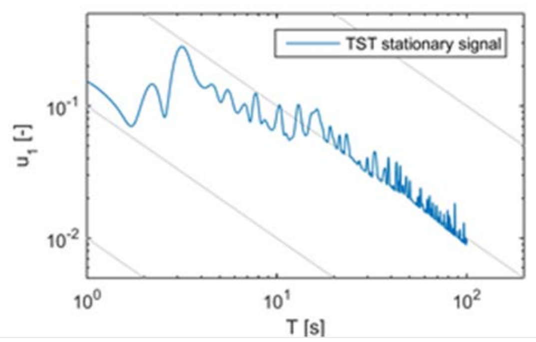

Fig. 4.4 – TST-B applied to original stationary signal.

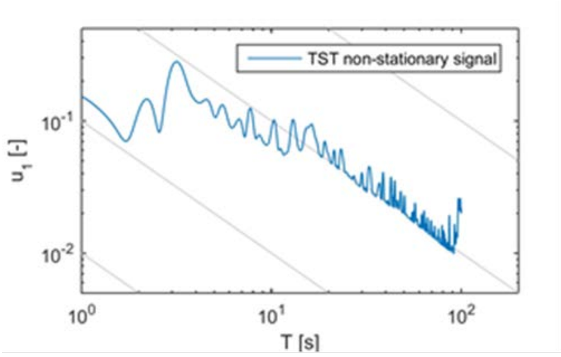

Fig. 4.5 – TST-B applied to non-stationary signal

Both TST results show a large range where u_1 decays with the inverse of T. The signal is stationary in this range. For small sections, approximately $T < 14$ s in this case, the values of u_1 fall below the trend of the stationary range. In this region, the section is becoming shorter than the longest oscillation periods in the stochastic process. In those cases the estimated standard deviation s underestimates the standard deviation of the stochastic process σ by a large factor due to a too short realisation length.

A sudden rise in u_1 of the non-stationary signal can be observed for T > 90 s. This rise is called a 'hockey stick'. Since the cumulative u_1 was calculated from the end of the signal, the hockey stick identifies a significant start-up transient for $t < t_{end}$ - 90.

The optimal section that provides the most accurate mean value of the stationary process is identified just left of the onset of the hockey stick. In this case around T equals 90 s. Since the cumulative u_1 was taken from the end of the signal, the last 90 s of the signal represent the optimal section. The first 10 s of the signal should be excluded. The non-stationary realisation and its optimal section are shown in Figure 4.6.

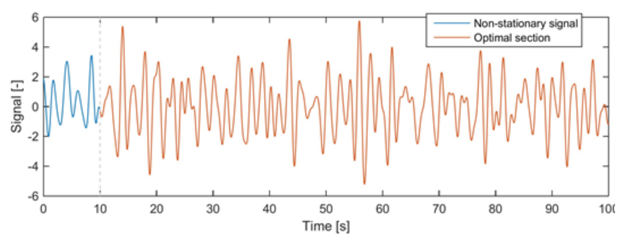

Figure 4.6 – Realisation with added start-up disturbance and the optimal section indicated.

The mean value of the complete signal is 0.031 and its estimated standard uncertainty u_1 is 0.022. Therefore the 95% confidence interval is constructed as (see Brouwer et al (2015b)):

$$u_1 = 0.022;\ k_{95} = 1.96;\ m = 0.031 \pm 0.044$$

k_{95} is the coverage factor for a confidence level of 95%, *m* is the sample mean.

The mean value of the optimal section is -0.0034 and its estimated standard uncertainty u_1 is 0.0105. Therefore the 95% confidence interval is constructed as:

u_1=0.0105; k_{95}=1.96; m=-0.003±0.021

The mean of the process μ which is zero, lies well within both estimated confidence intervals. Removing the start-up transient has reduced the error in the sample mean by a factor 10 and its corresponding uncertainty value by a factor of 2. The application of TST has increased the reliability of the mean value.

5. STEP BY STEP PROCEDURE

Although the mathematical processes behind the procedure can be challenging for 'non experts' the process of obtaining the TST is straightforward and is summarised in the following steps.

1. Obtain the time history ensuring that the sample rate is sufficient. For measurements in the laboratory or the field the sample rate must be at least twice (often more) the highest frequency in the time history.

2. Try to ensure that the time history is as long as practicable.

3. TST-A

a) Calculate the cumulative standard uncertainty for increasing sample lengths T using equations 5 and 6.

$$u_1 = \sqrt{\frac{1}{T}\int_0^T \left(1 - \frac{\tau}{t}\right) C_{xx,biased}(\tau) d\tau}$$

Where

$$C_{xx,biased}(\tau) = \left(1 - \frac{|\tau|}{T}\right) . C_{xx}(\tau)$$

b) Start at the beginning of the record with a small sample length T, calculate u_1 then increase the sample length T and recalculate u_1. Continue this process until T=total sample length.

c) Plot graph with T on the x-axis and u_1 on the y-axis, both with logarithmic scales.

d) Identify any region where the slope differs from -1. This indicates non stationary behavior in the measurement and this region should be removed from the mean estimation. Hockey sticks at large T indicate end effects.

4. TST-B

This procedure follows that of TST-A except the analysis begins at the end of the record with increasing values of T incorporating sample from the beginning of the record. Hockey sticks at large T indicate start-up effects.

6. REFERENCES

Lin, W.C., Oortmerssen, G., Davies, M.E., Faltinsen, O., Haimov, A.J., Hocquet, Ph., Della Logia, B., (1990), "Report of the Panel on Validation Procedures", Proceedings of the 18th ITTC.

Molloy. S., (2010), "Uncertainty Analysis of Ship Powering Prediction Methods", Amazon Distribution GmbH, Leipzig.

Brouwer J., Tukker, J. and M. van Rijsbergen (2013), "Uncertainty analysis of finite length measurement signals"; AMT'13.

J. Brouwer, J. Tukker, J. Dang and F.H. Lafeber (2015a), " Determining Statistical Properties of Quasi-steady Measured Signals" AMT'15.

J. Brouwer, J. Tukker and M. van Rijsbergen (2015b)," Uncertainty Analysis and Stationarity Test of Finite Length Time Series Signals"; AMT'15.

Bendat, J, Piersol, A.G., (2010) "Random data, analysis and measurement procedures" ,4th edition, John Wiley &Sons, Inc.,New Jersey.