ITTC Quality System Manual
Recommended Procedures and Guidelines

Procedure/Guideline

Computational Procedure for Instantaneous GZ Curve during Time-Domain Numerical Simulation in Irregular Waves

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-07	Loads and Responses
7.5-02-07-04	Stability
7.5-02-07-04.8	Computational Procedure for Instantaneous GZ Curve during Time-Domain Numerical Simulation in Irregular Waves

Disclaimer

Updated / Edited by	Approved
Stability in Waves Committee of 30th ITTC	30th ITTC 2024
Date: 08/2024	Date: 09/2024

Table of Contents

1. PURPOSE OF PROCEDURE 3
2. INTRODUCTION 3
  2.1 Change of GZ Curve in Waves 3
  2.2 Integration with Simulation Tools ... 4
  2.3 Factors, Influencing on the Instantaneous GZ Curve in Irregular Waves 4
3. CALCULATIONS 5
  3.1 Hydrostatic and Froude-Krylov Force 5
  3.2 Initialization of Calculation 5
  3.3 Balancing in Heave and Pitch 6
  3.4 Calculation of the Instantaneous GZ Curve 6
4. LIST OF SYMBOLS 7
5. REFERENCES 7
APPENDIX A. CALCULATION AND ANALYSIS EXAMPLE 8
  A.1. Input Data 8
  A.2. Quasi-static / Wave Pass Calculations 9
  A.3. Instantaneous GZ Curve in Irregular Waves 9
APPENDIX B. NEWTON-RAPHSON METHOD 10

Computational Procedure for Instantaneous GZ Curve during Time-Domain Numerical Simulation in Irregular Waves

1. PURPOSE OF PROCEDURE

This procedure describes a process for the computation of the instantaneous GZ curve in curve in irregular waves.

The instantaneous GZ curve may be useful for the interpretation of motion simulation results, especially episodes of large-amplitude roll angles.

2. INTRODUCTION

2.1 Change of GZ Curve in Waves

When a ship is sailing in longitudinal waves and the wave length is comparable to the ship length, the geometry of the submerged portion of the hull undergo significant changes. When a wave trough is located near midships, the upper part of the bow section is submerged, see Figure 1a. As many hull configurations include bow flare, the bow waterplane at the bow becomes wider. The upper part of the aft section may be also submerged, which would also contributing into making the instantaneous waterplane wider. The middle part of the hull has more shallow draft, but it does not affect waterplane so much as most of ships are wall-sided at the middle. As a result, the waterplane is larger and GZ is larger than in calm water.

When a wave crest is located near the midship section, the picture changes substantially, see Figure 1b. The bow section has a shallow draft and it makes the waterplane narrow as the bow section at lower draft is narrow. The same can be said about the aft section – it is narrow a lower draft. The draft is deep at the midship section, but it again does not affect the waterplane as most ship are wall-sided near midship section. As a result, the GZ curve is smaller when a wave crest is near the midship section.

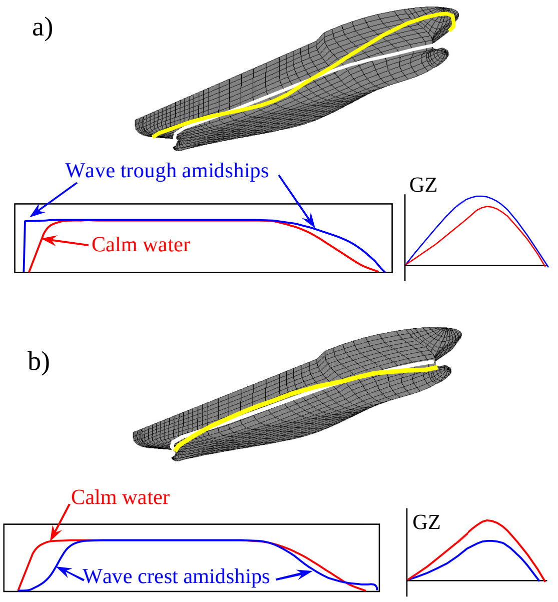

Figure 1: Changes in hull geometry (a) wave through is amidships (b) wave crest is amidships

Changes of the GZ curve in waves are root causes of two modes of stability failure: parametric roll resonance and pure loss of stability: see the IMO Interim Guidelines on the Second Generation Intact Stability Failure in MSC.1/Circ. 1627 and its Expiatory Notes MSC.1/Circ. 1651 as well as ITTC Recommended Procedure 7.5-92-07-04.3.

Quasi-static calculation of GZ curve in regular waves is available in most commercial software for ship hydrostatics. These calculations are very useful, as they characterize the degree of variability of stability in waves. However they do not provide a full description what is going on with stability at a particular instant of numerical simulations of ship motions in irregular waves. An instantaneous shape of the wave elevation is, indeed, a factor affecting stability. The other factor is forces, acting on a ship during the motion that are not present in a quasi-static setup, e.g. inertia.

Following d'Alembert principle, each instant of motion of a body can be treated as an equilibrium if inertia forces are considered. The instantaneous GZ curve in wave can be determined in the same way as in hydrostatics, if the all the forces acting on a ship are included in the equilibrium.

2.2 Integration with Simulation Tools

The GZ curve in waves can be computed at any instant of a simulation in waves by evaluating the roll restoring moment for variations in the ships roll angle. The general algorithm for the calculation of the instantaneous GZ curve in waves at an instant of time is as follows:

- The forces and moments acting on the ship are calculated at this instantaneous position
- The ship is heeled through a range of prescribed angles relative to its instantaneous position from the simulation
- The forces and moments acting on the ship are calculated at each heeled position.
- The ship is balanced by adjusting its vertical position and pitch angle relative to the mean free surface such that the total vertical force and pitch moment match those from the instantaneous position in the simulation.
- The instantaneous GZ value is determined from the net restoring moment in roll.

This algorithm requires the re-calculation of the relevant forces and moments for each angle of heel plus at each iteration of balancing in heave and pitch. As a result, the calculation of the instantaneous GZ curve may bear significant computation costs. At the same time, one does not really need the entire time history of the instantaneous GZ curve over the entire duration of a record. As the intended application is meant to aid in the interpretation of large roll angles, the instantaneous GZ curve is only needed for a brief, but specific time interval. Thus, these calculations should be considered as a part of post-processing.

2.3 Factors, Influencing on the Instantaneous GZ Curve in Irregular Waves

As the calculation of the instantaneous GZ curve is a part of a ship motion simulation tool, the forces to be included are limited to the forces available in the simulation tool. However, if there are options in calculation of these forces, as on the degrees of freedom, the following recommendations may be considered:

- Hydrostatic and Froude-Krylov forces: body-nonlinear formulation is required.
- As a minimum, the simulation is required to include heave, roll and pitch motions in order to perform proper evaluation of the body-nonlinear forces including restoring.
- Diffraction and radiation forces are desirable. There is indication that without the diffraction and radiation forces included, variation of the instantaneous GZ curve may be overestimated (Umeda et al. 2005). Note that the change of these forces with the heel angle variations may be difficult or very expensive to compute with some numerical methods.
- The effect of hull lift may be significant at high speed (Hashimoto and Umeda, 2004).
- If the configuration includes fins or other controls systems that provide both roll

damping and restoring, the change of these forces with the heel variation should be considered.
- Surge motion can affect the instantaneous position of the ship on the wave, which can further influence the periodic variation of GZ.

3. CALCULATIONS

3.1 Hydrostatic and Froude-Krylov Force

In a general case of 6 DoF calculations, a routine computing the hydrostatic and Froude-Krylov force can be presented as a 6-dimensional vector-value function $\vec{F}(\vec{X};t)$. Its argument is 6-dimensional state vector $\vec{X}$ and its parameters is time *t*. Following standard ship hydrodynamic notation, the components of the state vector $\vec{X}$ are surge ξ, sway η, heave ζ, roll ϕ, pitch θ and yaw ψ:

$$\vec{X} = (\xi, \eta, \zeta, \phi, \theta, \psi)^T \tag{1}$$

where, by default, a vector column is assumed and the superscript T stands for transposition. The function $\vec{F}(\vec{X};t)$ is constructed from components of principal vector and principal moment of hydrostatic and Froude-Krylov force:

$$\vec{F}(\vec{X};t) = (Fx, Fy, Fz, Mx, My, Mz,)^T \tag{2}$$

where *Fx*, *Fy* and *Fz* are the components of principal vector of hydrostatic and Froude-Krylov force in surge, sway and heave direction, respectively; *Mx*, *My* and *Mz* are the components of principal moment of hydrostatic and Froude-Krylov force about *x* axis (roll moment), y-axis (pitch moment) and z-axis (yaw moment), respectively. Two-letter symbols are used to avoid possible confusion with too many indexes

The hydrostatic and Froude-Krylov forces are usually computed in Earth-fixed or sliding coordinate system, where waves are defined. The hydrostatic and Froude-Krylov forces are inseparable under body-nonlinear formulation.

3.2 Initialization of Calculation

The GZ wave calculations at any instant of a simulation are initialized for the "current" time t_c with the instantaneous value of the state vector $\vec{X}$ and the corresponding value of the vector-valued function $\vec{F}(\vec{X};t)$:

$$\overrightarrow{Xc} = \vec{X}(t_c);\ \overrightarrow{Fc} = \vec{F}(\overrightarrow{Xc};t_c) \tag{3}$$

The GZ calculations start from the initial values $\overrightarrow{Xc}$ and $\overrightarrow{Fc}$ and are performed with positive and negative variations of the roll angle. A recommended increment of roll angle $\Delta\phi$ is 1 or 2 degrees; the index *j* is reserved here for roll angles:

$$\phi_j = \phi_c \pm j \cdot \Delta\phi \tag{3}$$

where $\phi_c = Xc_4$ is the "current" roll angle from the simulation.

The calculation of the instantaneous GZ value in waves starts from constructing a state vector for the first incremented roll (*j*=1). For the positive direction of roll angles:

$$\phi_1 = \phi_c + \Delta\phi \tag{4}$$

$$\vec{X}_{0,1} = (\xi_c, \eta_c, \zeta_c, \phi_1, \theta_c, \psi_c)^T \tag{5}$$

The first index of the state vector (5) indicates that is in an initial value before balancing in heave and pitch. Further the first index is used for a number of balancing iteration. The second index refer to the roll angle. The subscript c indicates that a value is taken from the "current" state vector $\overrightarrow{Xc}$.

3.3 Balancing in Heave and Pitch

The current value of the hydrostatic and Froude-Krylov forces, $\overrightarrow{Fc}$, corresponds to a dynamical equilibrium in terms of the d'Alembert principle. The incremental changes of the roll angle disturbs not only dynamical equilibrium in roll, in which it provides variations to the restoring moment, but due to hydrostatic coupling, it disturbs dynamic equilibrium in other degrees of freedom as well. As a result, after the roll is increment, other degrees of freedom should be re-balanced so that only the roll moment has changed. Such heave/pitch balancing is implicit in static heeling tests and nearly universal in the calculation of restoring curves in calm water.

In calm water, there is no hydrostatic coupling between vertical plane motions (heave, roll, pitch) and horizontal plane motions (surge, sway, yaw). Horizontal plane motions are in neutral equilibrium in calm water, i.e. do not have hydrostatic restoring. In waves, the body-nonlinear Froude-Krylov and hydrostatic forces, do, in principle, create coupling: change of underwater geometry due to vertical motion alters the hull's position in the wave pressure field and may change the horizontal plane components hydrostatic and Froude-Krylov force. The resulting coupling between vertical and horizontal plane motions for ship-like bodies is not believed to be large and can be neglected. As a result, balancing needs to be done only in heave and pitch.

The balanced condition for a roll angle ϕ_j (3) is expressed as follows:

$$\overrightarrow{Fb}\left(\overrightarrow{Xb}_j; t_c\right) = (Fc_3, Fc_5)^T \tag{6}$$

or

$$\overrightarrow{Xb}_j = \left(\xi_c, \eta_c, \zeta_b, \phi_j, \theta_b, \psi_c\right)^T \tag{7}$$

where ζ_b and θ_b are "balanced" heave and pitch values to be found.

The "balanced" condition can be expressed as zeros of the vector-valued function $\vec{F}^*$:

$$\vec{F}^*\left(\overrightarrow{Xb}_j; t_c\right) = (0,0)^T \tag{8}$$

where

$$\vec{F}^*\left(\overrightarrow{Xb}_j; t_c\right) = \overrightarrow{Fb}\left(\overrightarrow{Xb}_j; t_c\right) - (Fc_3, Fc_5)^T \tag{9}$$

Numerical solution of the balancing conditions can performed using Newton-Raphson method. For best performance, the initial value of the state vector can be taken from the previous heel angle, i.e.

$$\vec{X}_{0,j} = \overrightarrow{Xb}_{j-1} \tag{10}$$

A brief description of Newton-Raphson method of vector-valued function is given in Appendix B for easy reference.

3.4 Calculation of the Instantaneous GZ Curve

The calculation of the instantaneous GZ for the roll angle ϕ_j (3) is straight forward; it is a lever of a roll moment:

$$GZ\left(\phi_j\right) = \frac{F_4\left(\overrightarrow{Xb}_j; t_c\right)}{Fc_3} \tag{11}$$

where Fc_3 is the instantaneous heave force acting on the ship. Depending on the circumstances, it may be more informative to calculate the restoring arm with the ship weight W or present it with respect to the relative heel angle

$\phi_j - \phi_c$ rather than the total roll angle, but such usage should be clearly indicated.

Example of calculation is available in Appendix A.

4. LIST OF SYMBOLS

b	as a subscript or the the second symbol in the nomenclature means “balanced value”
c	as a subscript or the the second symbol in the nomenclature means “taken at the current instant of time”
i	index for iteration in balancing
j	index for increments in roll angles
$\mathbf{J}_F$	Jacobian matrix for the vector valued function $\vec{F}$
$\vec{F}$	Vector valued function, containing principal vector vector (components 1 through 3) and principal moment (components 4 through 6) of hydrostatic and Froude-Krylov force.
$\vec{F}^*$	Vector valued function for balancing, containing heave force and pitch moment
F_i	i-th component of vector-valued function $\vec{F}$; $i = 1, \dots 6$; the first three components are of the principal vector, while the last three components are of the principal moment of hydrostatic and Froude-Krylov force
Fx, F_1	Froude-Krylov force in surge
Fy, F_2	Froude-Krylov force in sway
Fz, F_3	Hydrostatic and Froude-Krylov force in heave
Mx, F_4	Hydrostatic and Froude-Krylov moment in roll
My, F_5	Hydrostatic and Froude-Krylov moment in pitch
Mz, F_6	Froude-Krylov moment in yaw
t	Time as an independent parameter
T	as a superscript, identifies transposition
$\vec{X}$	state vector, contains surge, sway, heave, roll, pitch and yaw
$\vec{X}_{0,j}$	Initial state vector, corresponding to j-th roll angle increment
$\Delta\phi$	Roll angle increment
ζ	Sway displacement
η	Sway displacement
θ	Pitch angle
ξ	Surge displacement
ϕ	Roll angle
ϕ_j	Roll angle, j-th increment
ψ	Yaw angle

5. REFERENCES

Belenky, V., K. Weems, W. M. Lin, V. Pipiras & T. Sapsis (2024) “Estimation of Probability of Capsizing with Split-Time Method,” Ocean Eng., Special Issue on Extreme Stochastic Dynamics of Ships Vol. 292, 116452, 27 p.

ITTC (2005) “The Specialist Committee on Stability in Waves, Final Report and Recommendations to the 24th ITTC,” Proc. of the 24th Int’l. Towing Tank Conf., Vol. II, pp. 369-407.

Hashimoto, H. & N. Umeda (2004) “Nonlinear Analysis of Parametric Rolling in Longitudinal and Quartering Seas with Realistic Modeling of Roll-restoring Moment”. J. of Marine Sci. and Tech., Vol. 9, pp. 117-126.

Shin, Y. S., V. L., Belenky, J. R. Paulling, K. M. Weems & W. M. Lin (2004) “Criteria for Parametric Roll of Large Containerships in Longitudinal Seas”, SNAME Trans. Vol. 112, pp. 14-47.

Spanos, D. & A. Papanikolaou (2009) “Benchmark Study on Numerical Simulation Methods for the Prediction of Parametric Roll of Ships in Waves,” Proc. 10th Int’l. Conf. on

Stability of Ships and Ocean Vehicles, St. Petersburg, Russia, pp. 627–636.

Spyrou, K. J., V. Belenky, A. Reed, K. Weems, N. Themelis & I. Kontolefas (2014) “Split-Time Method for Pure Loss of Stability and Broaching-To,” Proc. 30th Symp. Naval Hydrodynamics, Hobart, Tasmania, Australia.

Umeda, N., A. Munif & H. Hashimoto (2000) “Numerical Prediction of Extreme Motions and Capsizing for Intact Ships in Following/ Quartering Seas,” Proc. 4th Osaka Colloquium on Seakeeping Performance of Ships, Osaka, Japan, pp. 368-373.

Umeda, N., H. Hashimoto, G. Sakamoto & S. Urano (2005) “Research on Estimation of Roll Restoring Variation in Waves,” Proc. the Kansai Society of Naval Architects, Vol. 24, pp. 17-19.

Weems, K., & V. Belenky (2023) “Volume-base Reduced-Order Model for ship motions,” Ocean Eng., Special Issue on Extreme Stochastic Dynamics Ships, Vol. 289, 116214, 14 p.

Weems, K., V. Pipiras, and V. Belenky (2023a) “Multifidelity Fast Code for Direct Stability Assessment,” Proc. of 19th Intl. Ship Stability Workshop, Istanbul, Turkey, pp. 201-210.

Appendix A. CALCULATION AND ANALYSIS EXAMPLE

A.1. Input Data

This example for the procedure uses ITTC-A1 ship (Umeda et al. 2000), whose principal particulars are summarized in Table A1. This configuration was used in the ITTC benchmarking (ITTC, 2005) and SAFEDOR project (e.g. Spanos and Papanikolaou 2009). Roll decay data were available from the latter reference. The lines are shown in Figure A1.

Fast volume-based simulation tool (Weems, and Belenky 2023) was used to generate sample data. Simulation included 3 degrees of freedom: heave-roll-pitch. A body-nonlinear formulation was applied for hydrostatic and Froude-Krylov forces. Diffraction and radiation for heave and pitch and diffraction for roll was approximated from potential flow simulation tool LAMP (Large Amplitude Motion Program, Shin, et al 2003), while added mass and damping for roll was extracted from roll decay test of (Spanos and Papanikolaou 2009), see Weems et al. (2023a) for details.

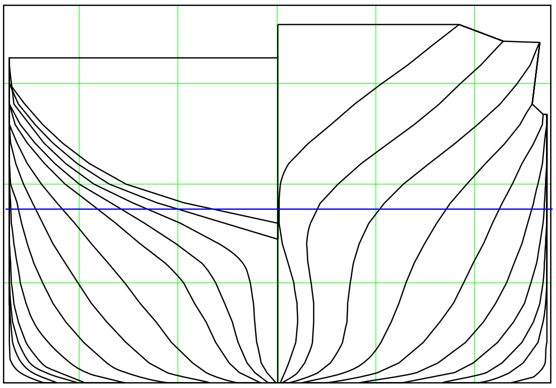

Figure A1 Lines of ITTC-A1 ship

Table A1 Principle Input Data

Length BP, m	150
Breadth, m	27.2
Draft amidships, m	8.5
KG , m	10.24
GM ,m	1.38
CB	0.667
CM	0.959
CW	0.786

A.2. Quasi-static / Wave Pass Calculations

Figure A2 shows results of quasi-static / wave pass calculation of the GZ curve in wave. The wave length was 150 m while wave height was 4 m.

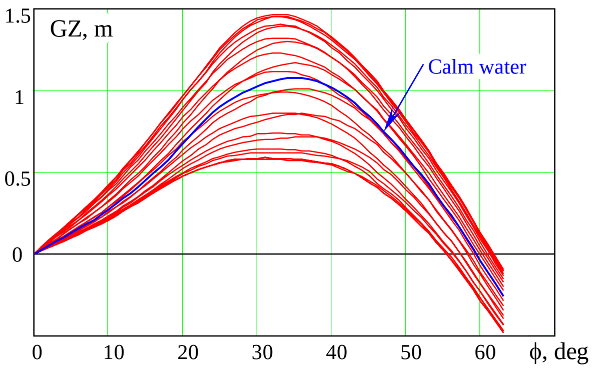

Figure A2 GZ curve in wave. Wave length 150 m, wave height 4 m

A.3. Instantaneous GZ Curve in Irregular Waves

To illustrate the dynamic approach for the instantaneous GZ curve calculation, a dataset from example to ITTC recommended procedure 7.5-02-07-04.6 (Rev. 1) was used.

The simulations were performed in wave environment, represented by long-crested irregular waves generated with Bretschneider spectrum recommended by ITTC 1978. The significant wave height was 7.5 m and modal period 14 s. The spectrum was discretized with 240 frequencies from 0.2 to 0.8 1/s. The time step was 0.5 s, with the ramping time of 10 s. Calculations we done for forward speed of 10 kt at stern quartering heading angle of 45 degrees. Duration of a record without self-repeating effect was 40 min.

The record #19 with a single exceedance of 40 degrees was used in this example, see Figure A3. Instantaneous GZ curve was computed for a time interval 810 s to 825 s where the exceedance occurs. The results are shown in Figure A4.

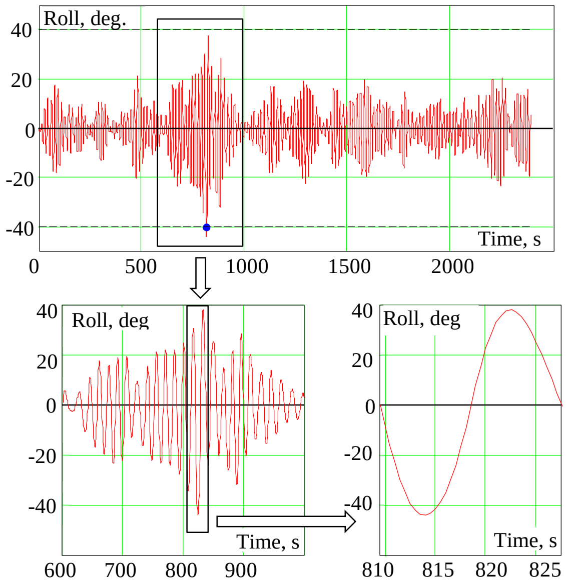

Figure A3 Record with exceedance 40 degrees at 812 s with two zoomed-in plots

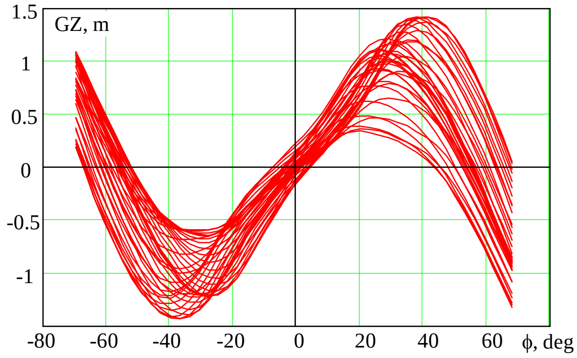

Figure A4 Instantaneous GZ curves computed in time interval 810-825 s.

Figure A4 shows significant variation of stability hinting that it may be the reason for the large roll angle around 814 s. However to perform a complete analysis of this large roll angle episode one may need to compute the instantaneous GZ curves during the previous roll period as well. Example of such analysis can be found in Spyrou et al. (2014) and Belenky et al. (2024).

Appendix B. NEWTON-RAPHSON METHOD

The Newton-Raphson method is intended for numerical solution of a system of equations that are continuous and differentiable. While its implementation is present in most of libraries of routines for numerical analysis, sometimes it has to be re-implemented in a ship motion numerical simulation tool due to licencing or compatibility reasons.

The objective of the method is to find zeros of a m-dimensional vector-valued function $\vec{F}(\vec{X})$ of a n-dimensional vector argument $\vec{X}$. The method is iterative, its i-th iteration is expressed as:

$$\vec{X}_i = \vec{X}_{i-1} - \left(\mathbf{J}_{\mathrm{F}}(\vec{X}_{i-1})\right)^{-1} \cdot \vec{F}(\vec{X}_{i-1})$$

where $i = 1, \dots, N$, and $\vec{X}_0$ is initial guess value. The index with variable with vector sign here refers to an iteration, $\vec{X}_i$ is i-th iteration on the vector $\vec{X}$. The iterations stop, upon reaching maximum number of iteration N or when the length of difference between two consecutive iterations fells below specified tolerance ε:

$\mathbf{J}_F(\vec{X})$ is a Jacobian matrix of the $\vec{F}(\vec{X})$ computed at $\vec{X}$:

$$\mathbf{J}_{\mathrm{F}}(\vec{X}) = \begin{pmatrix} \frac{\partial F_1}{\partial X_1} & \cdots & \frac{\partial F_1}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial X_1} & \cdots & \frac{\partial F_m}{\partial X_n} \end{pmatrix}$$

Index at a variable without vector sign stands for a number of component, so X_1 is the first component of vector $\vec{X}$.