ITTC Quality System Manual
Recommended Procedures and Guidelines

Procedure

Extrapolation for Direct Stability Assessment in Waves

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-07	Loads and Responses
7.5-02-07-04	Stability
7.5-02-07-04.6	Extrapolation for Direct Stability Assessment in Waves

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Table of Contents

1. PURPOSE OF PROCEDURE 3

2. INTRODUCTION 3

2.1 Formulation of the Problem 3

2.2 Theoretical Background 3

2.3 Envelope Peak-over-Threshold 4

2.4 Split-Time / Motion Perturbation.... 5

2.5 Extrapolation over Wave Height..... 6

2.6 Relation Between Probability and Time... 6

3. ENVELOPE PEAKS OVER THRESHOLD (EPOT)........................ 7

3.1 Data Requirement........................... 7

3.2 Data Preparation............................. 7

3.3 Fitting the Distribution 7

3.4 Assessment of Uncertainty............... 8

4. SPLIT-TIME / MOTION PERTURBATION METHOD (MPM) 9

4.1 General 9

4.2 Data Requirements 9

4.3 Data Preparation............................. 9

4.4 Fitting the Distribution................... 10

4.5 Assessment of Uncertainty 11

5. EXTRAPOLATION OVER WAVE HEIGHT 11

5.1 Data Requirements......................... 11

5.2 Data Preparation 12

5.3 Regression...................................... 12

5.4 Assessment of Uncertainty 13

6. LIST OF SYMBOLS 13

7. REFERENCES 14

APPENDIX A. CALCULATION EXAMPLE 15

A.1. Input Data 15

A.2. Results of EPOT Extrapolation 16

A.3. Results of Split-Time/ MPM Extrapolation 17

A.4. Results of Extrapolation over Significant Wave Height................. 18

A.5. Testing Extrapolation Method and Extrapolation Comparison Results 19

Extrapolation for Direct Stability Assessment in Waves

1. PURPOSE OF PROCEDURE

This procedure provides detailed guidance on the extrapolation of ship motion numerical simulation data in order to estimate the probabilities of large roll angle and capsizing in irregular waves. The Envelope Peak over Threshold (EPOT) method is described for the estimation of the rate of exceedance of large roll angles. The Split-time / Motion Perturbation method (MPM) is described for the estimation of the rate of exceedance of large roll angles or capsizing events. The Extrapolation over Significant Wave Height method is intended for the estimation of rate of exceedance of large roll angle and capsizing events. The procedure also highlights the limitations of these methods. Current procedure describes extrapolation for a given sea state, speed and heading.

2. INTRODUCTION

2.1 Formulation of the Problem

The complexity of the physical phenomena related to stability in waves makes numerical simulation the only tool that is suitable for detail assessment. Numerical simulations are, however, limited in the amount of data that can be practically generated.

Extrapolation allows the extension of existing numerical simulation data beyond observation. For example, if no exceedances of 40° were observed over 10 hr of simulation, but there were some 30° and 35° exceedances, extrapolation methods estimate the exceedance rate of 40° by using the existing data rather than running more simulations. The extrapolation carries more statistical uncertainty than an estimate based on direct direct observation. Extrapolation is the only technique capable of estimating probability of capsizing, as the latter is too rare to observe in realistic conditions.

The IMO second generation intact stability criteria allow the use of extrapolation as one of the tools for direct stability assessment (section 3.5.5 of MSC.1/Circ.1627).

2.2 Theoretical Background

Extrapolation is based on two theorems on extreme values. The first extreme value theorem (or Fisher-Tippett-Gnedenko theorem) proves that a distribution of the largest value in a sample has a limit in the form of a Generalized Extreme Value (GEV) distribution. The second extreme value theorem (or Pickands-Balkema-de Haan theorem) shows that the GEV distribution can be approximated by a Generalized Pareto Distribution (GPD) above a large-enough threshold. Extrapolation applications can be developed using both GEV and GPD distributions.

A brief overview of application of the first and the second extreme value theorems can be found in subsection 5.3.1 of Appendix 4 to the Explanatory Notes to the Interim Guidelines on the Second Generation Intact Stability Criteria (MSC.1/Circ.1652), while more details are available from Coles (2001).

The EPOT and Split-time /MPM methods use the extreme value theory, while the Extrapolation over Wave Height method has a different background. The latter method is based on the idea that stability failures are responses to groups of large waves. As the encounter probability of these groups scales with significant wave height, observation-estimated failure rate in higher sea states can be extrapolated to lower sea states. The Extrapolation over Wave Height Method is described in Section 5.1 of Appendix 4 to the Explanatory Notes (MSC.1/Circ.1652).

2.3 Envelope Peak-over-Threshold

Peak-over-threshold (POT) method is a generic extrapolation method based on GPD (Pickands, 1975). A key feature of the POT extrapolation is that it can capture the nonlinearity of the large amplitude response, such as that caused by the changes in the restoring at large roll angles and in waves.

The tail ($y>u$) of *any* distribution can be approximated with a GPD above a sufficiently large threshold. The GPD is defined by three numbers – a shape parameter ξ, a scale parameter σ, and threshold value u – and has the following form for $y>u$:

$$\mathrm{pdf}(y) = \begin{cases} \frac{1}{\sigma}\left(1+\xi\frac{y-u}{\sigma}\right)^{-\left(1+\frac{1}{\xi}\right)} & \xi \neq 0 \\ \frac{1}{\sigma}\exp\left(-\frac{y-u}{\sigma}\right) & \xi = 0 \end{cases} \tag{1}$$

$$\mathrm{cdf}(y) = \begin{cases} 1-\left(1+\xi\frac{y-u}{\sigma}\right)^{-1/\xi} & \xi \neq 0 \\ 1-\exp\left(-\frac{y-u}{\sigma}\right) & \xi = 0 \end{cases} \tag{2}$$

The objective of the present application is to estimate a rate of exceedance $\hat{\lambda}(c)$ of a target value $c>u$ above the threshold u:

$$\hat{\lambda}(c) = \hat{\lambda}(u)\widehat{\mathrm{cdf}(c)} \tag{3}$$

where $\hat{\lambda}(u)$ is the rate of upcrossing of the threshold u, estimated directly form the time series.

For the application of GPD, three parameters must be found: shape ξ, scale σ and threshold u. The scale parameter σ is positive, while the shape parameter ξ can be either positive or negative. A negative shape parameter imposes a limitation on the expressions in parenthesis of equations and formally introduces a right bound to the distribution:

$$\mathrm{pdf}(y) = 0, \quad if\ \ y > u - \frac{\sigma}{\xi} \ \text{ and } \xi < 0 \tag{4}$$

The shape parameter defines the type of tail: heavy, exponential, or light, as shown in Figure 1.

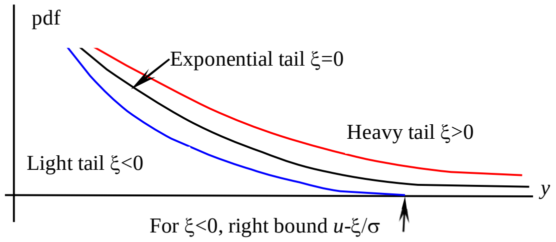

Figure 1 Types of tails per GPD approximation

The roll restoring arm ($\overline{GZ}$) curves of most ships have a limited range of stability, leading to the appearance of an unstable equilibria at the angle of vanishing stability, as well a maximum value of $\overline{GZ}$. This configuration leads to a heavy tail after the maximum of the $\overline{GZ}$ curve, which switches to a light tail in the immediate vicinity of the angle of vanishing stability. Figure 2 illustrates the configuration, see Belenky et al. (2019) for details.

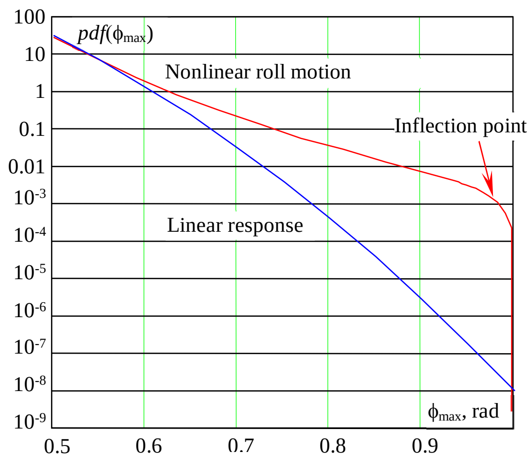

Figure 2 Configuration of a trail of roll peak distribution

The standard POT method is only applicable for independent data points, while the roll motions of a ship are correlated because of the ship's inertia, correlated wave excitation, and "memory" in the hydrodynamic forces. The application of POT, therefore, requires an extraction of independent points from the time history, a process known as "de-clustering."

Fitting an envelope to the time history of the roll motion, as illustrated in Figure 3, is a convenient way to de-cluster the data, as the peaks of the envelope of the roll response are sufficiently far from each other to provide the necessary independence. The use of an envelope to de-cluster the roll motion provides the additional letter in the acronym of the method, so POT becomes EPOT – Envelope Peaks over Threshold

Declustering with the envelope uses a property of the relatively narrow banded spectrum of roll motion that is usually correct for beam, following and stern-quartering seas. As the roll spectrum grows wider in head and oblique seas, the envelope peaks may need to be checked for decorrelation as recommended in Section 3 of this document.

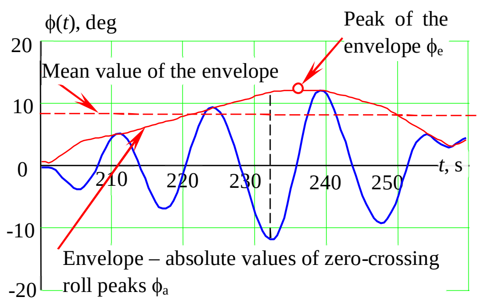

Figure 3 De-clustering using an envelope

2.4 Split-Time / Motion Perturbation

For the cases where physics may change with a further increase of roll angles, (e.g. water enters on deck), the split-time motion perturbation method (MPM) may be applicable. The method is also capable for estimating the rate of capsizing events.

The estimation of the rate of exceedances or capsizings requires an additional step: calculation of a metric of likelihood with MPM. The roll rate is perturbed until target roll angle or capsizing event is observed (see Figure 4). The difference between the roll rate at upcrossing and the roll rate when the target roll angle or capsizing is observed is the metric of likelihood, as this difference indicates "the distance to trouble."

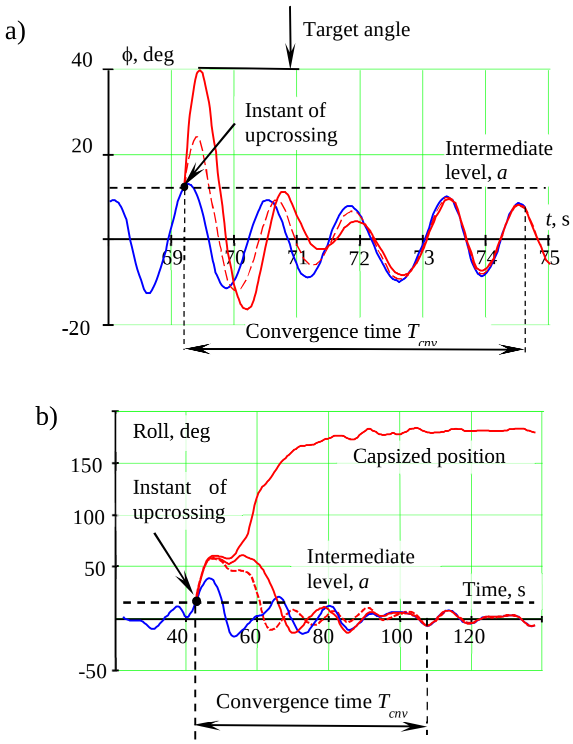

Figure 4 Motion perturbation for computing the metric likelihood for (a) large roll angle (b) capsizing

The metric of likelihood is a random number showing how likely large roll angle or capsizing is at a given instant of time. It has been demonstrated that the tail of the MPM metric of likelihood is likely to be exponential when applied to capsizing. An algorithm for the calculation of the metric of likelihood and its extrapolation are given in Section 4 of this document.

2.5 Extrapolation over Wave Height

The main idea of extrapolation over wave height is that a usually rare stability failure may become observable through numerical simulation when wave height is exceptionally large. For such waves, the rate of events (either exceedances of large roll angle or capsizing) can be estimated from numerical simulation through direct counting, as described in ITTC recommended procedure 7.5-02-01-10 "Frequency of Random Events". Performing these estimation over a series of very large significant wave heights generates data for an extrapolation towards more moderate conditions where stability failures cannot be observed in a practical numerical simulation, see Söding and Tonguć (1986).

The reason why the approach works is in the group structure of waves. A ship responds to a group of large waves. A height of the largest wave in a group follows a conditional Rayleigh distribution and thus have an exponential tail (Figure 1). The waves around the largest wave in a group are also expected to be large, so the conditional Rayleigh distribution is applicable to the size of the wave group and the severity of the ship response.

The dependence of the rate estimates over the significant wave height is presented as:

$$\ln \hat{\lambda} = \hat{A}_{\lambda} + \hat{B}_{\lambda}/H_s^2 \quad (5)$$

where $\hat{A}_{\lambda}$ and $\hat{B}_{\lambda}$ are estimated through regression over several large-wave simulations. Additionally, one can estimate a reciprocal value of an average mean time before a failure:

$$\ln \hat{T}_f = \hat{A}_T + \hat{B}_T/H_s^2 \quad (6)$$

where $\hat{A}_T$ and $\hat{B}_T$ are also estimated through regression over several large-wave simulations. More information on the theoretical background of the method is available from Shigunov (2023) and Anastopoulos and Spyrou (2023). A comparison between the extrapolation over wave height and other extrapolation method can be found in Wandji (2022). Figure 5 provides a general illustration of the method.

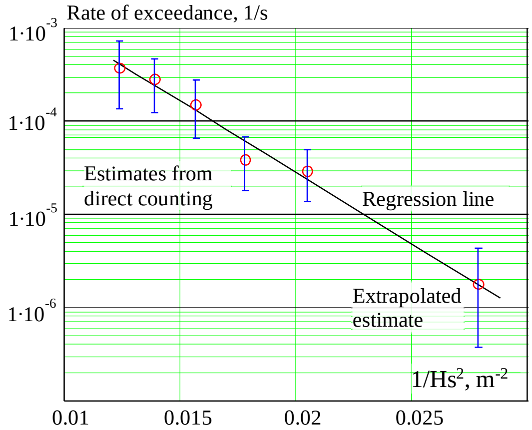

Figure 5 Concept of extrapolation over wave height

2.6 Relation Between Probability and Time

Finally, the relation between the probability and time needs to be mentioned. If there is a requirement for the estimation of a probability of exceedance of a certain roll angle or a capsizing, a time of exposure, T_e must be specified. The probability that at least one exceedance (or capsizing) will occur during the time T_e can be estimated as:

$$\hat{P}(T_e) = 1 - \exp\left(-\hat{\lambda}(c) T_e\right) \quad (7)$$

where $\hat{\lambda}(c)$ is an estimate of exceedance or the event rate. More background information is available from Section 2 of IMO document SDC 8/INF.2 (IMO, 2021).

3. ENVELOPE PEAKS OVER THRESHOLD (EPOT)

3.1 Data Requirement

The recommended combined length of time histories of roll motion is 40 hours. While the bulk of statistical validation was performed for 100 hours time histories (Campbell et al. 2023a), a successful test for 40 hours is described in IMO document SDC 7/INF.3 (IMO, 2019), subsection 5.3.3.

It may be possible to run the EPOT extrapolation for shorter time. It will require a favourable comparison of the extrapolation based of the shorter simulation with the extrapolation using at least 40 hours of the simulation time history.

The simulation time histories should include sufficient information on nonlinearity of the roll motion. At least 5% of the envelope peaks should exceed a half of the angle of maximum of the GZ curve in calm water. Caution has to be exercised when applying the method to oblique heading, while this requirement may be slightly relaxed in stern-quartering seas. The general theoretical background for the method is available from Campbell et al. (2023).

3.2 Data Preparation

It is convenient to present roll motion time history data as a set of independent records. The length of each record should be about 30 minutes.

Zero-crossing peaks (min/max value between sequential zero value crossings) are extracted from each record. The absolute values of these peaks ϕ_a form the envelope as shown in Figure 3.

The mean value for the envelope is estimated for each record. Mean-crossing peaks of envelope ϕ_e are collected for further processing.

In the case of oblique seas, when the spectrum of roll motion is widening, an additional check for data independence needs to be performed. Time is recorded together with each peak of roll motion and retained for the mean-crossing envelope peaks ϕ_e.

A decorrelation time T_d (time duration to reach independence) is estimated as described in the subsection 4.2.2 of the ITTC Procedure 7.5-02-01-08 (see Figure 1). Estimation of auto-covariance function $\hat{R}$ is described in the subsection 4.2.2 of the above procedure.

The time between the envelope peaks should be more than the decorrelation time T_d. If one or several peaks of the envelope are located closer then T_d , the largest among them is used for further processing.

3.3 Fitting the Distribution

The tail of the distribution of a large roll angle is supposed to be heavy (see Figure 2). When the shape parameter $\xi > 0$ and threshold value $u = \sigma/\xi$, the GPD is equivalent to a Pareto distribution with scale $y_m = \sigma/\xi$ and shape $\alpha = 1/\xi$:

$$\mathrm{pdf}(y) = \frac{\alpha y_m^{\alpha}}{y^{\alpha+1}} \tag{8}$$

The conditional probability of exceedance of a target value y associated with dynamic stability failure is expressed as:

$$\mathrm{P}(Y > y | Y > u) = \left(\frac{u}{y}\right)^{\alpha} = \left(\frac{y}{u}\right)^{-\frac{1}{\xi}} \tag{9}$$

Here, the threshold u does not have to be the same as in the GPD case. A method for finding the threshold and estimating the shape parameter is proposed in Campbell et al. (2023), which is based on Beirlant et al. (2004), Dupuis and Victoria-Feser (2006), and Mager (2015).

To extrapolate with equation (7), the threshold is found from applicability considerations so only one parameter needs to be fitted.

The input data for fitting consists of N independent envelope peaks ϕ_e (Figure 3). After the independence of the peaks has been established, there is no need to keep track of the number of records and the time when a peak has been recorded. The method is applied to a sample sorted in descending order – a.k.a. order statistics:

$$Y = \mathrm{sort}_{\mathrm{desc}}(\phi_e) \tag{10}$$

The Hill estimator provides the shape parameter ξ – for the case of Pareto $\xi > 0$:

$$\hat{\xi}_k = \frac{1}{k}\sum_{i=1}^{k} \log\left(\frac{Y_i}{Y_k}\right) \tag{11}$$

where the index k refers to the number of upper order statistics used in the estimation. Mager (2015) suggests the first index $k = \max(40, 0.02N)$, while the last (largest) value for the index is taken as $0.2N$.

The threshold u is found by an index that corresponds to a minimum of the mean squared prediction error function:

$$\hat{\Gamma}(k) = \frac{1}{\hat{\xi}_k^2 \cdot k}\sum_{i=1}^{k} \frac{\left(\log\left(\frac{Y_{i-1}}{Y_{k-1}}\right) + \hat{\xi}_k \log\left(\frac{i}{k+1}\right)\right)^2}{\left(\sum_{j=i}^{k} j^{-2}\right)} + \frac{2}{k^2}\sum_{i=1}^{k} \frac{\left(\log\left(\frac{i}{k+1}\right)\right)^2}{\left(\sum_{j=i}^{k} j^{-2}\right)} - 1 \tag{12}$$

Once the index k corresponding to a minimum of $\hat{\Gamma}$, is found, the threshold is set as:

$$u = Y_k \tag{13}$$

The extrapolated estimate of the exceedance rate of target value c can be computed as:

$$\hat{\lambda}(c) = \hat{\lambda}(u)\left(\frac{c}{u}\right)^{-1/\hat{\xi}} \tag{14}$$

where $\hat{\lambda}(u)$ is the rate of upcrossing of threshold u, estimated as:

$$\hat{\lambda}(u) = \frac{k}{T_{tot}} \tag{15}$$

where T_{tot} is the total time of all the records available.

3.4 Assessment of Uncertainty

The uncertainty of an extrapolation coming from a finite volume of the data sample can be evaluated with a confidence interval containing the true value with confidence probability β.

The confidence interval for the extrapolated value is computed assuming a normal distribution for the estimate of the shape parameter $\hat{\xi}$. Its variance estimate is expressed as:

$$\widehat{Var}(\hat{\xi}) = \frac{\hat{\xi}^2}{k} \tag{16}$$

The boundaries of the confidence interval of the estimate are:

$$\hat{\xi}_{up,low} = \hat{\xi} \pm K_\beta \sqrt{\widehat{Var}(\hat{\xi})} \tag{17}$$

where K_β is a half of a non-dimensional confidence interval computed as a normal quantile of $0.5(1+\beta)$. For $\beta=0.95$, $K_\beta=1.96$.

The number of upcrossings of the threshold k has a binomial distribution with the estimate of a parameter

$$\hat{p} = \frac{k\,\Delta t}{T_{tot}} \tag{18}$$

where Δt is time increment used in the simulation. The confidence interval for $\hat{\lambda}(u)$ is computed using normal approximation for binomial distribution. The variance of the number of upcrossings k is estimated as:

$$\hat{V}_k = \frac{T_{tot}}{\Delta t}\hat{p}(1-\hat{p}) \tag{19}$$

Then the confidence interval for $\hat{\lambda}(u)$ is coumputed as:

$$\hat{\lambda}_{up,low}(u) = \frac{N_{aU} \pm K_\beta\sqrt{\hat{V}_k}}{T_{tot}} \tag{20}$$

Boundaries for the extrapolated value are computed through the lower and upper boundaries of the upcrossing rate estimate $\hat{\lambda}_{low,up}(u)$ and the shape parameter estimate $\hat{\xi}_{low,up}$:

$$\begin{cases} \hat{\lambda}_{low}(c) = \hat{\lambda}_{low}(u)\left(\frac{c}{u}\right)^{-1/\hat{\xi}_{low}} \\ \hat{\lambda}_{up}(c) = \hat{\lambda}_{up}(u)\left(\frac{c}{u}\right)^{-1/\hat{\xi}_{up}} \end{cases} \tag{21}$$

Equations (21) contain a product of the boundaries of two estimates. If the desired confidence probability for the entire extrapolated estimate $\hat{\lambda}(c)$ is to be $\beta = 0.95$, then the confidence probabilities for each estimate $\hat{\lambda}(u)$ and $\hat{\xi}$ must be set as:

$$\beta_1 = \sqrt{\beta} = \sqrt{0.95} = 0.975 \tag{22}$$

In order to account for the difference in the confidence probability, K_β is set to 2.236 in equations (16) and (20).

4. SPLIT-TIME / MOTION PERTURBATION METHOD (MPM)

4.1 General

The primary application area for the Split-Time / Motion Perturbation Method (MPM) is estimating the probability of complex and rare physical phenomena in which the physics of the problem changes with the extreme response, such as that caused by capsizing in dead ship conditions or pure loss of stability in stern quartering / following seas. The general theoretical background of the method is available from Belenky et al. (2024) while statistical validation is described by Weems et al. (2023).

4.2 Data Requirements

The recommended combined length of time histories of roll motion is 40 hours. While the bulk of statistical validation was performed for 100 hours time histories (Weems et al. 2023), a successful test for 40 hours is described in IMO document SDC 7/INF.3, subsection 5.3.4 (IMO, 2019).

It may be possible to run the split-time/MPM method using shorter simulation time. It will require a favorable comparison of the extrapolation based of the shorter simulation with the extrapolation using at least 40 hours of the simulation time history.

There is no requirement on how large the roll motions should be. The treatment of nonlinearity is included in the computation of the metric of likelihood.

4.3 Data Preparation

The metric of likelihood is computed at the instant of upcrossing of an intermediate roll level a.

The value of the intermediate level a is chosen based on the consideration of computational efficiency. If the level is set too high, there will be too few upcrossings available for the metric computations, so additional simulation time may be required. If the level is set too low, there will be many upcrossings leading to dependent metric values. Some of these values will be rejected during the de-clustering procedure, which will decrease computational efficiency. 7-10 upcrossings per an independent record of 30 minutes long was found to be acceptable.

The evaluation of the metric consists of the following steps.

1. Compute the time instant of up crossing t_{Ui} and the roll rate at the instant of upcrossing $\dot{\phi}_{Ui}$ using linear interpolation between the roll angle and rate data points, respectively. Similarly compute values of heave, pitch and their derivatives at the instant of upcrossing.
2. Compute a perturbed solution with initial conditions $\phi_0 = a$ and $\dot{\phi}_0 = \dot{\phi}_{Ui} + \Delta\dot{\phi}$ and the observed values for pitch and heave, computed at the previous step. If the target angle or capsizing is not observed and the perturbed solution converges with the unperturbed solution (see Figure 4), the convergence time T_{cnv} is defined when the difference between perturbed and unperturbed solution does not exceed a given value for a given number of points.
3. The next perturbation is carried out for $\dot{\phi}_0 = \dot{\phi}_{Ui} + 2\Delta\dot{\phi}$, keeping the rest of the initial conditions the same. The procedure is repeated for $3\Delta\dot{\phi}$, $4\Delta\dot{\phi}\ldots$, until the target angle or capsizing is observed as shown in Figure 4 for $\dot{\phi}_0 = \dot{\phi}_{Ui} + m\Delta\dot{\phi}$, where m is the number of iterations.
4. Once the target angle or capsizing is observed for $\dot{\phi}_0 = \dot{\phi}_{Ui} + m\Delta\dot{\phi}$, the critical roll rate is computed as $\dot{\phi}_{Ci} = \dot{\phi}_{Ui} + (m-1)\Delta\dot{\phi}$, convergence time T_{cnv} for the penultimate iteration is recorded for further use in the de-clustering procedure.
5. Metric for the i-th upcrossing is computed as:

$$y_i = 1 + \dot{\phi}_{Ui} - \dot{\phi}_{Ci}\,; \quad i = 1,\ldots,N_U \tag{23}$$

where $\dot{\phi}_{Ci}$ is the critical roll rate calculated for the i-th upcrossing, and $\dot{\phi}_{Ui}$ is the roll rate observed at the i-th upcrossing, N_U number of observed upcrossings.

If 6-DOF simulations are used for perturbation, the decorrelation time T_d (time duration to reach independence, see subsection 4.2.2 of the ITTC Procedure 7.5-02-01-08) is used instead on the convergence time T_{cnv}. As shown by Belenky et al. (2024), 6-DOF perturbations normally do not converge to the initial time history.

The up crossings of a level usually comes in groups, the metric values are clustered, meaning that the metric values within a same cluster may be dependent. A de-clustering procedure is used to ensure independence of the collected metric values. The cluster is defined as a group of the metric values y_i, corresponding to up crossings that are closer than respective convergence time durations $\{y_i\}_{i=b_j}^{i=e_j}$, where b_j and e_j are indexes of the beginning and end of j-th cluster. The declustered values are determined as the maximum value within each cluster:

$$x_j = \max\left(\{y_i\}_{i=b_j}^{i=e_j}\right), j = 1\ldots N \tag{24}$$

4.4 Fitting the Distribution

As demonstrated in Glotzer et al. (2024), the independent large values of the metric (21) can be approximated with an exponential tail. Fitting a tail involves finding a threshold, after which the exponential approximation is applicable. Finding this threshold (not to be confused with intermediate level for upcrossings) is carried out with the prediction error criterion (Mager 2015).

To use the prediction error technique, the data needs to be sorted in descending order. A mean squares prediction error function is defined as (Mager 2015):

$$\hat{\Gamma}(k) = \frac{1}{\hat{\gamma}_k^2}\sum_{i=1}^{k}\left(\frac{k+1}{i}-1\right)^{-1}\left(x_{i,k}+\hat{\gamma}_k\log\left(\frac{i}{k+1}\right)\right)^2 + \frac{2}{k}\sum_{i=1}^{k}\left(\frac{k+1}{i}-1\right)^{-1}\left(\log\left(\frac{i}{k+1}\right)\right)^2 - 1, \quad (25)$$

where k is an index corresponding to a candidate threshold, while $\hat{\gamma}_k$ is exponential distribution parameter estimated on the subset of the data points indexes from 1 to k.

$$k \in [\max(40, 0.02N); 0.2N] \quad (26)$$

$$\hat{\gamma}_k = \frac{1}{k}\sum_{j=1}^{k}(x_j - x_k) \quad (27)$$

The resulting threshold u (not to be confused with intermediate level a, used earlier) is found where the mean squares prediction error function experiences a global minimum.

The extrapolated estimate of the capsizing rate is computed as

$$\hat{\lambda} = \hat{\lambda}(u)\exp\left(-\frac{1-u}{\hat{\gamma}}\right) \quad (28)$$

where the estimate of the parameter $\hat{\gamma}$ corresponds to the chosen threshold u, $\hat{\lambda}(u)$ is the rate of upcrossing of threshold u, estimated with equation (15).

4.5 Assessment of Uncertainty

The uncertainty of the estimation of the capsizing rate is driven by the finite volume of the data available for extrapolation. The confidence interval for the extrapolated value is computed assuming a normal distribution for the estimate of the parameter $\hat{\gamma}$. Its variance estimate is expressed as:

$$\widehat{Var}(\hat{\gamma}) = \frac{\widehat{Var}(x-u)}{n} \quad (29)$$

where $\widehat{Var}(x-u)$ is the variance estimated for the points above the threshold u, while n is the volume of sample above the threshold u. The boundaries of the confidence interval of the estimate are:

$$\hat{\gamma}_{up,low} = \hat{\gamma} \pm K_\beta\sqrt{\widehat{Var}(\hat{\gamma})} \quad (30)$$

where K_β is a half of a non-dimensional confidence interval computed as a normal quantile of $0.5(1+\beta_2)$. β_2 is the confidence probability for the parameter, computed as $\beta_2 = \sqrt{\beta}$, while β is the confidence probability, accepted for the entire extrapolated estimate.

Boundaries for the extrapolated estimate are computed through the lower and upper boundaries of the upcrossing rate estimate $\hat{\lambda}_{low,up}(u)$ (see equation 20), and the parameter estimate $\hat{\gamma}_{up,low}$:

$$\begin{cases} \hat{\lambda}_{low}(c) = \hat{\lambda}_{low}(u)\exp\left(-\frac{1-u}{\hat{\gamma}_{low}}\right) \\ \hat{\lambda}_{up}(c) = \hat{\lambda}_{up}(u)\exp\left(-\frac{1-u}{\hat{\gamma}_{up}}\right) \end{cases} \quad (31)$$

Similar to equations (21), equation (31) contain a product of the boundaries of two estimates. If the desired confidence probability for the entire extrapolated estimate $\hat{\lambda}(c)$ is to be $\beta = 0.95$ then the confidence probabilities for each estimate $\hat{\lambda}(u)$ and $\hat{\gamma}$ must be set as shown in equation (20)

5. EXTRAPOLATION OVER WAVE HEIGHT

5.1 Data Requirements

The data requirements for extrapolation over wave height are essentially the same as the requirements for direct counting, as described by ITTC recommended procedure 7.5-02-01-10.

Data are prepared for all combinations of speeds and headings for all the loading conditions that are of interest for the analysis.

5.2 Data Preparation

The data for extrapolation are prepared by running numerical simulation for at least three sea conditions where the significant wave height is large enough to collect a sample of sufficient-volume of stability failure events, as described by ITTC recommended procedure 7.5-02-01-10 "Frequency of Random Events". The modal period should be the same for the simulations.

Results are presented in a form of array of failure rates $\hat{\lambda}_i; i = 1, \dots, K$, estimated for $K \geq 3$ different large significant wave heights H_{si}. The range of these significant wave heights should be at least 2 m: $|H_{sK} - H_{s1}| \geq 2\,m$. Boundaries of confidence interval for the rate estimate are also presenting as arrays: upper $\hat{\lambda}_{Ui}$ and lower $\hat{\lambda}_{Li}$ boundaries, respectively.

If the extrapolation will be performed on the mean time before the event, the results are presented as an array of the respective values $\hat{T}_{fi}$, estimated for each of the large significant wave heights. The upper and lower boundaries of the confidence interval are $\hat{T}_{Ui}$ and $\hat{T}_{Li}$, respectively. Other relevant data are the number of encountered stability failures presented in the form of an array $N_{fi}; i = 1, \dots, K$.

5.3 Regression

The data described in the subsection 5.2 are used to estimate coefficients $\hat{A}_\lambda$ and $\hat{B}_\lambda$ for equation (5) or $\hat{A}_T$ and $\hat{B}_\lambda$ for equation (6). To perform linear regression, a matrix of predictors is constructed as:

$$\boldsymbol{X} = \begin{pmatrix} 1 & H_{s1}^{-2} \\ \dots & \dots \\ 1 & H_{sK}^{-2} \end{pmatrix} \tag{32}$$

The coefficients $\hat{A}_\lambda$ and $\hat{B}_\lambda$ are estimated as:

$$\left(\hat{A}_\lambda, \hat{B}_\lambda\right)^T = (\boldsymbol{X^T X})^{-1} \boldsymbol{X^T} \vec{\hat{L}}_\lambda \tag{33}$$

where subscript T is a transposition sign and $\vec{\hat{L}}_\lambda = \left(\ln\hat{\lambda}_1, \dots, \ln\hat{\lambda}_K\right)^T$ is K-dimensional vector of responses in terms of rate estimates, see subsection 5.2.

If the extrapolation is performed for the mean time before the event, the coefficients $\hat{A}_T$ and $\hat{B}_T$ are estimated as

$$\left(\hat{A}_T, \hat{B}_T\right)^T = (\boldsymbol{X^T X})^{-1} \boldsymbol{X^T} \vec{\hat{T}}_F \tag{34}$$

where $\vec{\hat{T}}_F = \left(\hat{T}_{f1}, \dots, \hat{T}_{fK}\right)^T$ is a K-dimensional vector of responses in terms of estimates of mean times before the events, see subsection 5.2.

The actual extrapolation for the significant wave height of interest H_s is performed with formula (5) for the event rate estimate or with formula (6) for the estimate of the mean time before the event.

Alternatively, extrapolation can be preformed with formulae from section 5.1.3 of Appendix 4 of MSC.1/Circ. 1652:

$$\hat{\lambda} = \exp\left(\textstyle\sum_{i=1}^K b_i \ln\hat{\lambda}_i\right), \tag{35}$$

where i a counter for the K data points, available from direct counting $(i = 1, \dots K)$ and coefficients b_i are computed as

$$b_i = \frac{H_S^{-2}}{\sum_{i=1}^K H_{si}^{-2}} + \left(1 - \frac{K \cdot H_S^{-2}}{\sum_{i=1}^K H_{si}^{-2}}\right) \times \frac{H_{si}^{-2} \cdot \sum_{i=1}^K H_{si}^{-2} - \sum_{i=1}^K H_{si}^{-4}}{\left(\sum_{i=1}^K H_{si}^{-2}\right)^2 - K \cdot \sum_{i=1}^K H_{si}^{-4}} \tag{36}$$

where H_s is a significant wave height for which extrapolation is is performed. These coefficients were derived with the least squares method and have a meaning of statistical weights, so $\sum_{i=1}^K b_i = 1$.

5.4 Assessment of Uncertainty

The uncertainty of the extrapolated estimate is quantified by propagation of the uncertainty of the data points $\hat{\lambda}_i$ through formula (5), having in mind that the latter is essentially a linear combination. The variance of the rate estimate from direct counting is:

$$\hat{V}_{\lambda i} = \frac{1}{N_i} \tag{37}$$

where N_i is a number of observations for i-th significant wave height. As these observations are independent, their covariance matrix contains only diagonal terms:

$$\widehat{\mathrm{Cov}}_{\lambda} = \begin{pmatrix} \hat{V}_{\lambda 1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & \hat{V}_{\lambda K} \end{pmatrix} \tag{38}$$

The covariance matrix for regression coefficients $\hat{A}_\lambda$ and $\hat{B}_\lambda$ is computed as:

$$\widehat{\mathrm{Cov}}_{AB\lambda} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\widehat{\mathrm{Cov}}_{\lambda}\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1} \tag{39}$$

$\boldsymbol{X}$ is a matrix of predictors, see equation (32). The variance of the extrapolated estimate as a function of the significant wave height, is computed as:

$$\hat{V}_{\lambda}(H_s) = (1, H_s^{-2})\widehat{\mathrm{Cov}}_{AB\lambda}(1, H_s^{-2})^T \tag{40}$$

A notional number of points that would be observed for the significant height H_s is related to the variance (40) as:

$$\hat{N}_e = \frac{1}{\hat{V}_{\lambda}(H_s)} \tag{41}$$

Alternatively, this number can be computed with coefficients b_i from equation (36):

$$\hat{N}_e = \left(\sum_{i=1}^{K}\frac{b_i^2}{N_i}\right)^{-1} \tag{42}$$

The boundaries for the confidence interval are computed with the chi-squared distribution as:

$$\hat{\lambda}_{low} = \frac{0.5\hat{\lambda}\cdot\chi^2_{0.5(1-\beta),2\hat{N}_e}}{\hat{N}_e} \qquad \hat{\lambda}_{up} = \frac{0.5\hat{\lambda}\cdot\chi^2_{0.5(1+\beta),2\hat{N}_e}}{\hat{N}_e} \tag{43}$$

6. LIST OF SYMBOLS

A_λ, B_λ	Regression parameters for extrapolation of exceedance rate over wave height
A_T, B_T	Regression parameters for extrapolation of time-to-failure over wave height
Cov	Covariance matrix
c	Target for extrapolation
cdf	Cumulative density function
H_s	Significant wave height
L_λ	Natural logarithm of failure rate
N	Total number of independent data points
$\hat{P}(T_e)$	Estimate of probability of at least one exceedance or capsizing occurs during the time of exposure
pdf	Probability density function
T_d	Time of decorrelation, s
T_e	Time of exposure, s
T_{tot}	Total time of available simulation, s
T_F	Time before failure event, s
u	Threshold value
$Var()$	Variance operator
X	Matrix of predictors
β	Confidence probability
γ	Parameter of exponential distribution
λ(c)	Exceedance or capsizing rate s^{-1}
λ(u)	Exceedance of a threshold u, s^{-1}
ξ	Shape parameter of Generalized Pareto distribution or Pareto distribution
σ	Scale parameter of Generalized Pareto Distribution
ϕ	Roll angle, deg
ϕ_a	Roll amplitude (absolute values of roll peaks), deg
$\dot{\phi}$	Roll rate, rad/s

$\hat{\square}$ Estimate (“hat” above a symbol)
$\square_{up}$ Upper boundary of confidence interval
$\square_{low}$ Lower boundary of confidence interval

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Appendix A. CALCULATION EXAMPLE

A.1. Input Data

Examples for the extrapolation procedures use the ITTC-A1 ship (Umeda et al. 2000), whose principal particulars are summarized in Table A1. This configuration was used in the ITTC benchmarking (ITTC 2005) and SAFEDOR project (e.g. Spanos and Papanikolaou 2009). Roll decay data were available from the latter reference. The lines are shown in Figure A1.

A fast, volume-based time-domain simulation tool (Weems and Belenky 2023, Weems et al. 2023a) was used to generate the sample data. Simulation included 3 free degrees of freedom (heave-roll-pitch) with constant course and speed. While these 3 degrees of freedom may be insufficient for direct assessment for all modes of failure under the Second Generation IMO Stability Criteria per MSC.1/Circ. 1627, this simplified simulation is capable of producing the data to serve as an example for extrapolation.

A body-nonlinear formulation was applied for hydrostatic and Froude-Krylov forces. Diffraction and radiation for heave and pitch and diffraction for roll was approximated from the potential flow simulation tool LAMP (Large

Amplitude Motion Program), while added mass and damping for roll was extracted from roll decay test of Spanos and Papanikolaou (2009), see Weems et al (2023) for details.

Table A1 Principle data of ITTC-A1 ship

Length BP, m	150
Breadth, m	27.2
Draft amidships, m	8.5
KG , m	10.24
GM ,m	1.38
CB	0.667
CM	0.959
CW	0.786

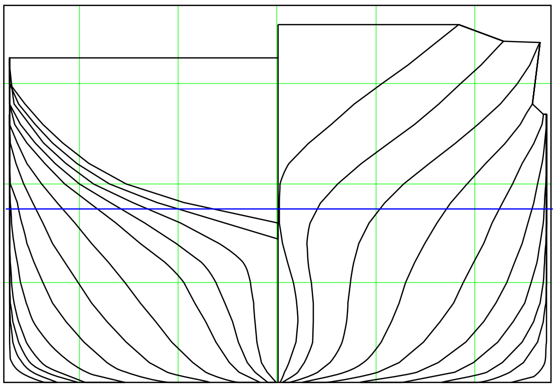


Figure A1 Lines of ITTC-A1 ship

The wave environment was represented by long-crested irregular waves generated with Bretschneider spectrum recommended by ITTC 1978. The significant wave height was 6 m and modal period 14 s. The spectrum was discretized with 240 frequencies from 0.2 to 0.8 1/s. The time step was 0.5 s, with a ramp-up time of 10 s. Calculations were done for forward speed of 10 kt at stern quartering heading angle of 45 degrees. Duration of a record without self-repeating effect was 40 min.

A.2. Results of EPOT Extrapolation

The total volume of the sample of motion data for EPOT extrapolation was 150 hours. While only about 40 hours is needed for EPOT, the dataset was extended for more meaningful comparison with the two other extrapolation methods.

A sample record is shown in Figure A2 (a). Figure 2A (b) illustrates constructing the envelope, selecting the mean-crossing values of the envelope and identifying independent data points as mean-crossing peaks of the envelope. There were total of 64082 zero-crossing peaks in 225 records, 29832 peaks exceeded the mean of envelope at 10.2 degrees. Using the envelope-based de-clustering procedure, 3568 peaks were found to be independent.

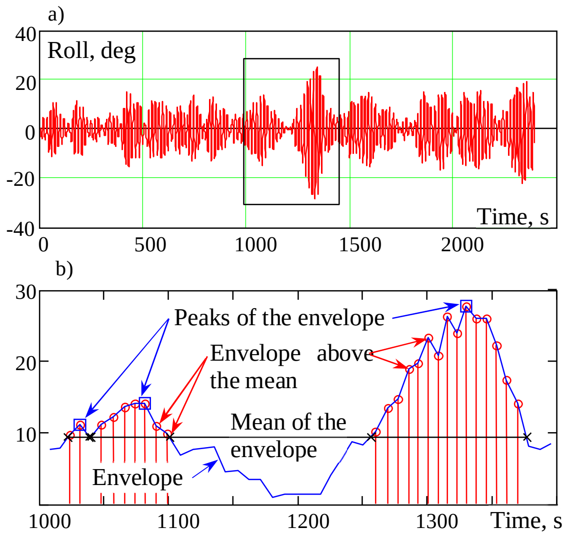


Figure A2 Sample record (a) envelope, its mean and peaks (b)

The first index to search for the threshold ($k = \max(40, 0.02N)$) were taken as 40, while the last index ($0.2N$) equals 714, leaving 675 potential thresholds. The mean squared prediction error function is shown in Figure A3.

The index corresponding to the minimum error value was found to be 643, corresponding to the angle of u=27.5 deg. The number of points available for fitting the tail is k =72.

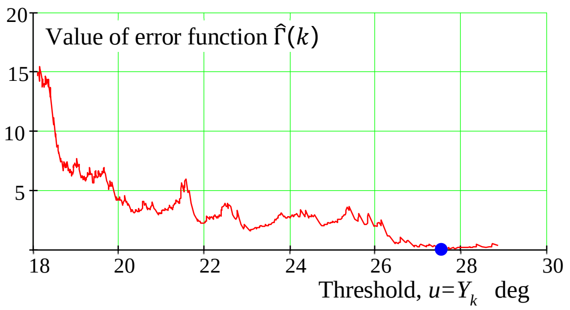


Figure A3 Mean squares prediction error function for Pareto tail

The shape parameter for the Pareto distribution was estimated as $\hat{\xi} = 0.0788$ using equation (11). The rate of upcrossing through the level u is estimated as $\hat{\lambda}(u) = 1.34 \cdot 10^{-4}\ s^{-1}$ with equation (15). The final result, which is the rate of exceeding of 40 degrees, is $\hat{\lambda}(c = 40) = 1.15 \cdot 10^{-6}\ s^{-1}$.

The Assessment of uncertainty requires the calculation of variances of the shape parameters of Pareto distribution using formula (16): $\widehat{Var}(\hat{\xi}) = 8.62 \cdot 10^{-5}$ and variance of the the number of upcrossing using formula (19) $\hat{V}_k = 71.995$. The final result is shown in Figure A8.

A.3. Results of Split-Time/ MPM Extrapolation

Split-Time / MPM extrapolation was performed for the same simulation dataset as EPOT, first for the 40 degrees exceedance rate and then capsizing rate.

The intermediate threshold was chosen at 20 degrees, resulting in 2699 upcrossings over all the 225 records. The de-clustering procedure produced 561 independent values of the metric. Calculations were carried out for both exceedance of 40 degrees and capsizing events.

For the choice of the threshold with the Prediction Error Criterion, the mean squares prediction error function is shown in Figure A4. Other intermediate numerical results are given in Table A2

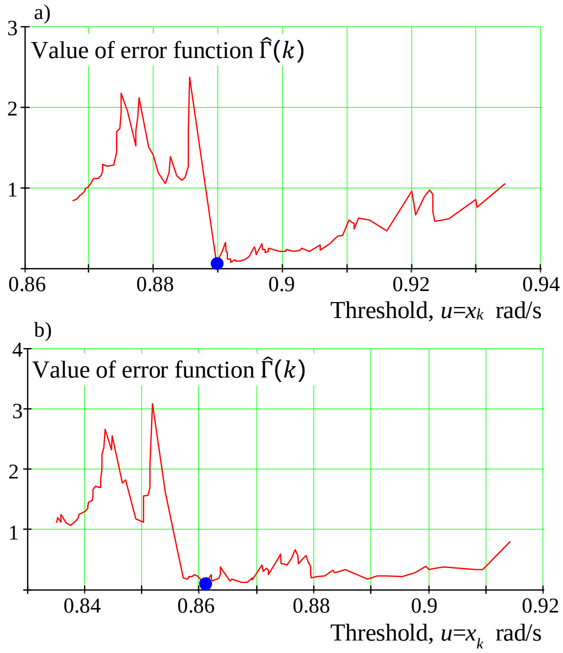


Figure A4 Mean squares prediction error function for exponential tail for (a) exceedance of 40 degrees (b) capsizing

Table A2 Intermediate Results of Fitting Exponential Tail

Parameter	Exceedance of 40 deg	Capsizing
Threshold, u	0.890	0.861
Available points	68	66
Parameter $\hat{\gamma}$ rad/s	0.024	0.027
Variance of $\hat{\gamma}$ (rad/s)^2	7.10 10^{-6}	9.049 10^{-6}
$\hat{P}(y > 1)$	0.011	5.743 10^{-3}
Rate estimate $\hat{\lambda}$, s^{-1}	1.337 10^{-6}	7.049 10^{-7}

The final result for the exceedance of 40 degrees is included in Figure A8, while Figure A5 shows the extrapolated estimate for the capsizing rate.

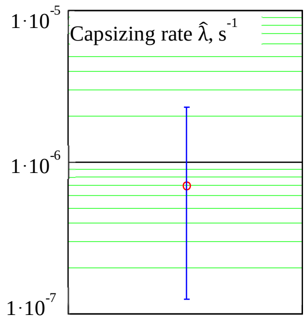


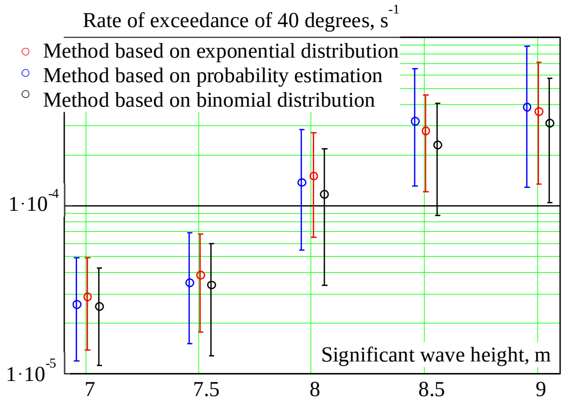
Figure A6 Estimate for Rate of Capsizing

A.4. Results of Extrapolation over Significant Wave Height

For the extrapolation over significant wave height, simulation were run for four sea states as shown in the table A3. The total number of 40-minutes records was 300, representing 200 hours of simulation. The number of records for each significant wave height was selected so as to observe 10 to 12 exceedances of 40 degrees angle to both port and starboard sides.

The rate of exceedance was estimated with all three methods described in the ITTC recommended procedure 7.5-02-01-10 "Frequency of Random Events". Numerical results are placed in Table A3, while a graphic depiction is presented at Figure A7.

Regression coefficients where calculated with equation (36) for all three methods. The notional number of observations, expressing the uncertainty of extrapolation and computed with equation (44) or (45), is the same for all three methods. Finally the extrapolated estimates were computed with equation (5). All these quantities are placed in Table A4. The extrapolated estimates with their confidence interval are also included in Figure A8.



Note that the estimation were made with these three methods for exactly same significant wave heights; the points were shifted for the convenience of presentations.

Figure A7 Estimation of rate of exceedance of 40 degrees angel by direct counting

Table A3 Parameters and Intermediate Results for Extrapolation over Significant Wave Height

Significant wave height	7.0	7.50	8.0	8.5	9.0
Records	150	100	25	15	10
Time, hrs.	100	66.7	16.7	10	66.7
Events	11	10	10	12	11
Method based on exponential distribution					
Rate, s^{-1}	$2.9\cdot10^{-5}$	$3.9\cdot10^{-5}$	$1.5\cdot10^{-4}$	$2.8\cdot10^{-4}$	$3.7\cdot10^{-4}$
Upper, s^{-1}	$4.9\cdot10^{-5}$	$6.8\cdot10^{-5}$	$2.7\cdot10^{-4}$	$4.5\cdot10^{-4}$	$7.1\cdot10^{-4}$
Lower, s^{-1}	$1.4\cdot10^{-5}$	$1.8\cdot10^{-5}$	$6.5\cdot10^{-5}$	$1.2\cdot10^{-4}$	$1.3\cdot10^{-4}$
Method based on probability estimation					
Rate, s^{-1}	$2.6\cdot10^{-5}$	$3.5\cdot10^{-5}$	$1.4\cdot10^{-4}$	$3.2\cdot10^{-4}$	$3.8\cdot10^{-4}$
Upper, s^{-1}	$4.9\cdot10^{-5}$	$6.9\cdot10^{-5}$	$2.5\cdot10^{-4}$	$6.5\cdot10^{-4}$	$8.8\cdot10^{-4}$
Lower, s^{-1}	$1.2\cdot10^{-5}$	$1.5\cdot10^{-5}$	$5.4\cdot10^{-5}$	$1.3\cdot10^{-4}$	$1.3\cdot10^{-4}$
Method based on binomial distribution					
Rate, s^{-1}	$2.5\cdot10^{-5}$	$3.4\cdot10^{-5}$	$1.2\cdot10^{-4}$	$2.3\cdot10^{-4}$	$3.1\cdot10^{-4}$
Upper, s^{-1}	$4.2\cdot10^{-5}$	$5.9\cdot10^{-5}$	$2.2\cdot10^{-4}$	$4.1\cdot10^{-4}$	$5.7\cdot10^{-4}$
Lower, s^{-1}	$1.1\cdot10^{-5}$	$1.3\cdot10^{-5}$	$3.4\cdot10^{-5}$	$8.7\cdot10^{-5}$	$1.0\cdot10^{-4}$

Table A4 Intermediate Results and Final for Extrapolation over Significant Wave Height

Method	Exponential distribution	Probability estimation	Binomial distribution
Intercept $\hat{A}_\lambda$	-3.49	-3.10	-3.77
Slope $\hat{B}_\lambda$	-351.0	-377.0	-344.4
Number $\hat{N}_e$	3.02	3.02	3.02
Estimate of rate, $\hat{\lambda}$, s^{-1}	$1.775\cdot10^{-6}$	$1.282\cdot10^{-6}$	$1.621\cdot10^{-6}$
Upper boundary, s^{-1}	$4.264\cdot10^{-6}$	$3.079\cdot10^{-6}$	$3.895\cdot10^{-6}$
Lower boundary, s^{-1}	$3.688\cdot10^{-7}$	$2.663\cdot10^{-7}$	$3.368\cdot10^{-7}$

A.5. Testing Extrapolation Method and Extrapolation Comparison Results

The results of extrapolation with all three methods are shown in Figure A8. The extrapolation over wave height was performed with three method for the estimation of rate of exceedance of 40 degrees, following ITTC recommended procedure 7.5-02-01-10 "Frequency of Random Events". While EPOT and Split-time/MPM show narrower confidence interval compare to the extrapolation over significant wave height, the upper boundaries of confidence intervals are very close. As these upper boundaries are the final results of direct stability assessment, all three methods applied in this example appear to be equivalent.

Following the principles described in ITTC recommended procedure 7.5-02-01-10 "Statistical Validation of Extrapolation Methods for Time Domain Numerical Simulation of Ship Motions", the extrapolation procedure can be tested against direct observation of the exceedance events in a large-volume sample simulated with the same method and for the same conditions, used for extrapolated estimates.

A large-volume sample was generated to test the extrapolation procedures. It contains 5000 records of 40 minutes each, equating 3219 hours. The sea state parameters were consistent with those described in the section A1 of this Appendix, i.e. significant wave height was 6 m at 14 s modal period. Totally there were 11 exceedance events observed. The estimates exceedance rate was performed in with three method, following ITTC recommended procedure 7.5-02-01-10 "Frequency of Random Events". The results are placed in Table A5 and shown in Figure A8.

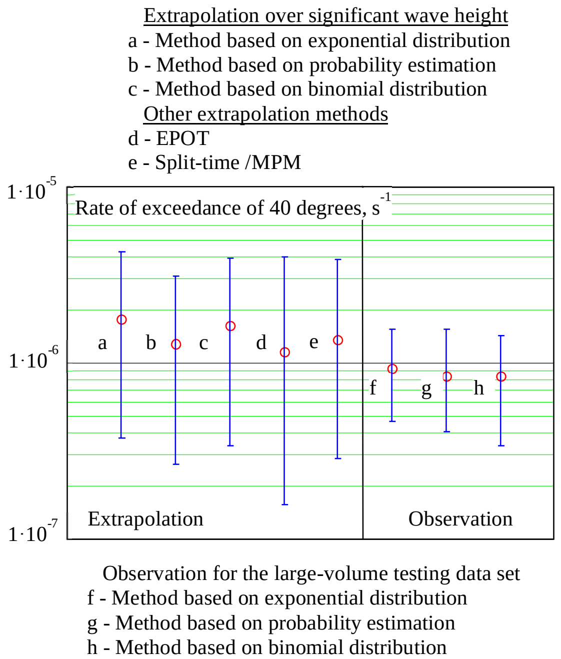


Figure A8 Comparison and testing extrapolation methods

Table A5 Estimates of Rates of Exceedance from Large-Volume Test Sample

Method	Exponential distribution	Probability estimation	Binomial distribution
Estimate of rate, $\hat{\lambda}$, s^{-1}	$9.21 \cdot 10^{-7}$	$8.38 \cdot 10^{-7}$	$8.37 \cdot 10^{-7}$
Upper boundary, s^{-1}	$1.54 \cdot 10^{-6}$	$1.54 \cdot 10^{-6}$	$1.42 \cdot 10^{-6}$
Lower boundary, s^{-1}	$4.6 \cdot 10^{-7}$	$4.02 \cdot 10^{-7}$	$3.35 \cdot 10^{-7}$

All the confidence interval of the extrapolation estimates completely contain the results of direct observations, making this particular test successful.