ITTC Quality System Manual
Recommended Procedures and Guidelines

Guideline

Uncertainty Analysis – Example for Horizontal-Axis Current Turbines

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Uncertainty Analysis - Example for Horizontal-Axis Current Turbines

1. PURPOSE OF THE UNCERTAINTY ANALYSIS

The purpose of the guideline is to provide guidance on the application of uncertainty analysis to the small-scale testing of a current turbine following the ITTC Procedure 7.5-02-07-03.9, “Model Tests for Current Turbines”. The small-scale testing of a current turbine, while similar to the testing of a propulsion device or pump, focuses on the measurement of energy extraction from the flowing water in contrast to the addition of energy to a hydro environment by a pump or propulsor.

The uncertainty analysis should be performed following the ITTC Procedures (7.5-02-01-01, “Uncertainty Analysis in EFD, Uncertainty Assessment Methodology,” and 7.5-02-01-02, “Uncertainty Analysis in EFD, Guideline for Towing Tank Tests”. In addition, the ITTC procedures and guidelines relevant to the uncertainty in powering and resistance testing would be examples of the application of an uncertainty procedure to a marine turbomachinery device: ITTC Procedures (7.5-02-03-01.2, “Propulsion, Performance Uncertainty Analysis, Example for Propulsion Test,” 7.5-02-02-02, “Uncertainty Analysis, Example for Resistance Tests,” and 7.5-02-03-02.2, “Uncertainty Analysis Example for Open Water Test”.

2. INTRODUCTION

Unlike a standard powering test of a propulsor or pump, the measurement of the power extraction of a current turbine will be strongly dependent on the power take-off (PTO) used in the model scale testing. The model scale PTO design may not be representative of the full-scale PTO design and may add a level of uncertainty to the measurement of model-scale power extraction and the prediction of full-scale power extraction potential.

The device TRL (Technology Readiness Level, on a scale from 1 to 9) or stage of development can determine the type of testing performed, full device or sub-component testing, as well as the degree or extent of the uncertainty analysis required. This then defines what analyses should be performed and recommended levels of uncertainty that should be targeted. In addition, the target audience of the test (Developer, Investor or Certifying body) can also dictate the level of uncertainty that needs to be achieved and what needs to be analysed. In general, the goal of a current turbine device is power extraction from the hydro environment. An uncertainty analysis of a turbine device must be focused on the uncertainty in the power measurement and all contributing sources of error in that measurement. Section 3 will provide a Summary of the Error Contributions that must be accounted for in a current turbine test.

It is expected that the standard Design of Experiments, Montgomery (2012), uncertainty analysis methodologies defined by Coleman and Steele (1999), Taylor *et al.* (1993), and the various ITTC procedures and guidelines referenced throughout this guideline will be followed relative to:

A) performing the analysis,
B) designing/planning the test program,
C) interpreting the uncertainty analysis with respect to the device or sub-component performance, and

D) proper presentation of the uncertainty analysis results.

An example of an uncertainty analysis applied to a current turbine test is provided.

3. SUMMARY OF ERROR CONTRIBUTIONS

3.1 Scaling

The unsuitable use of scaling laws when designing model scale systems for small-scale testing can contribute to errors in device function that can propagate into the uncertainty in power extraction. Geometric, dynamic and kinematic similitude should be attempted in the design of a small-scale device for testing when possible. Particular care should be taken with hydroelastic behaviours, especially regarding the flexibility of slender structures such as long blades. When complete similitude cannot be achieved due to governing physics of the device (Froude scaling vs. Reynolds scaling) or manufacturing limitations, the impact of mixed scaling on the device performance and error contribution should be assessed. Improper scaling can arise in the device sub-component function as well. Improper scaling of the sub-components of a device, such as wings or PTO, can also impart added errors due to improper function of the sub-component.

The primary/dominant scaling factor in the operation of a current turbine should be the Reynolds number (Re) with the correct velocity and length reference used. The Re provides a measure of the state of the flow regime over the device (laminar, transitional or turbulent) and this flow regime can have significant impact on the steady and unsteady device performance.

The proper use of Re scaling can be important in device performance and the resulting errors that may be encountered in model scale testing. Different characteristics of a device may scale differently with Re depending on the velocity and length scale used.

The impact of unsuitable Re scaling on uncertainty can be difficult to assess due to the complexities of laminar to turbulent transition and the relative impact of transition on a component function. For example, an open, multi-blade turbine (similar to a wind turbine), designed to operate in a turbulent Re regime will extract lower power than expected from a given inflow if tested in a lower laminar Re flow regime.

The magnitude of the reduced power extraction will depend on the blade design and may be difficult to quantify a-priori leading to a biased interpretation of the small-scale test results. This can manifest itself as a bias error in the full-scale prediction of power extraction. This error can be difficult to quantify in an uncertainty analysis for full-scale prediction. It is recommended that Re scaling be adhered to in small testing to avoid these possible errors.

In reality, *Re* matching can be difficult to achieve in small-scale model testing. This often leads to necessary flow speeds higher than can be accommodated in a facility, and increased model loads that could result in uncertainty in performance assessments due to unrealistic blade deflections. In some cases, these increased loads could lead to catastrophic model failures.

Re-scaling may also lead to facility-related inflow and boundary constraints causing unscaled turbulent flow characteristics or changes in flow blockage. It is important to recognise when Re scaling cannot be achieved and to what extent it is mismatched between model and full-scale for proper interpretation of the model-scale test results. The primary impact of improper Re scaling is the determination of the flow regime on the model – laminar, transitional or fully turbulent.

If the proper full-scale flow regime (laminar versus turbulent) cannot be maintained at model scale testing, techniques can be used to artificially trip the boundary layer on transition-sensitive components to attempt to control the transition location on the component. This may be necessary for traditional turbine blade designs with a laminar flow leading edge geometry over a substantial percentage of the leading-edge chord. The performance characteristics of these blade designs are known to be sensitive to the location of transition on the blade and thus the Reynolds number (Re_C) based on blade chord. The application of a boundary trip on the blade can be used to control transition and produce blade performance characteristics at the lower test Re_C that are more in line with those that would be achieved at higher Re_C. Tripping can be difficult and requires much care in data interpretation. Methodologies in the ITTC Procedure 7.5-01-01-01 “Ship Models,” for hull models, could be used as initial guidance for techniques to design boundary layer trip features for control of boundary layer transition in model tests. The best method would be to test at increasing Reynolds numbers to verify an asymptotic behaviour.

Facility operating characteristics can also compromise transition control. High free-stream turbulence levels or high-frequency model vibration could initiate early transition on a laminar flow-controlled surface. Care must be taken to identify, control and document facility/model operating characteristics such as free stream turbulence intensity levels and power spectrum or model vibration characteristics if these may be relevant to component boundary layer transition and overall device function.

3.2 PTO Sources of Error

The PTOs main purpose is to convert mechanical power extraction from the hydro device, turbine, to electrical energy. The PTO is a component system comprised of a drive train and power generation modules. Model-scale device testing must include some form of PTO modelling. In tests of model current devices, the PTO can be represented by direct electrical power generation, by mechanical/hydraulic/pneumatic loading or by using a speed or torque control drive. In all cases, friction associated with bearings and seals must be carefully assessed in order to minimise the impact on the measured power. Fine control of the static/initial torque is essential for experiments with small-scale models.

Uncertainty due to the PTO can be characterised in two categories. The first is associated with the small-scale device test, and the second is in predicting scale-up performance based on the small-scale testing. Errors in small-scale device tests typically result from frictional effects in bearings and seals, instrument use (resolution, accuracy, etc.) and scaling of the full-scale PTO to model scale. The measure of power extraction requires that the mechanical device used to extract power from the flow is restricted in its motion such that the fluid has to work on the device to induce that motion. This requires the PTO to provide resistance to the mechanical device motion through some form of a mechanical, hydraulic or pneumatic load.

Scale-up performance prediction uncertainty usually occurs when PTO design variants are introduced into the model scale relative to the full-scale PTO.

3.3 Model Errors

Model errors can be grouped into three categories: manufacturing, structural and functional.

3.3.1 Manufacturing

Manufacturing errors result from the inability to properly scale a model due to manufacturing limitations. Typical examples include edge geometries, surface finish and general manufacturing tolerances. The function of a laminar flow turbine blade can depend on both the leading-edge geometry and the blade surface finish. If the full-scale device is designed to have a critical leading-edge radius or a tight surface finish, the model scale must have an appropriately scaled edge radius and surface finish. Both the leading-edge geometry and the surface finish can have a strong impact on the location of the boundary layer transition on the blade, and this can impact blade performance. A model scale surface finish that is hydraulically rough at model scale but smooth at full scale can bias model-scale testing due to differences in boundary layer characteristics over the device components.

Edge geometry and surface roughness can also impact flow shedding on components and flow induced vibration and noise. Careful analysis of the model scale geometry and flow regime over the model scale components must be performed to assess any potential impact on model scale device performance.

Manufacturing tolerances may need to be relaxed at model scale due to manufacturing limitations. If a device has critical, tight clearances between components that could impact device performance, scaling these clearances down to model scale could result in difficulties in assembly or in maintaining these scaled clearances due to operational or thermal effects. A ducted turbine may have gap clearances defined to be in a specific range to optimise turbine performance in the field. Scaling these clearances down in a model scale may produce a rotor to duct fit that cannot be maintained during operation due to normal operating vibration/movement of the mating components or due to thermal expansion/contraction of the different mating components.

The scaling of tip gap flows in a ducted device can be sensitive to the physical gap size. Small gaps can increase the rotational resistance of the device due to increased viscous losses associated with the model scale gap flows. Large gaps can impact device performance by increasing the gap bleed flow and reducing turbine blade lift over the outer 10% span of the blade due to increased blade-tip flow leakage. These flow-induced characteristics due to improper gap scaling can introduce bias errors in the power production of the device, overall device loading/drag and component loading such as drive shaft torque.

Assessing the impact of manufacturing limitations on model performance and quantifying the level of bias error that can be introduced can be difficult. Experience and sound engineering judgement may be the only approach to quantifying a level of uncertainty or error in these situations. Computational techniques may be able to be used to bracket a level of error due to surface finish discrepancies. If surface finish is suspected of altering boundary layer transition on a model-scale blade relative to that encountered on the full-scale blade, computational modelling could be performed to assess blade performance (lift and drag) as a function of boundary layer transition location and surface roughness. Increased device drag or vibration of sub-components interfering with the movement of mating components can increase shaft torque and bias power estimation. Vibrational effects can be challenging to assess in common tare or zeroing tests.

The bias errors that shaft seals and bearings can introduce at model scale can be addressed through proper tare testing. Typically, a rotating bare hub test can be performed to identify the torque required to overcome frictional losses associated with the shaft bearings and seals. These torque estimates can then be used to correct the measured device torque for these frictional losses.

3.3.2 Structural

Structural errors arise from improperly scaling the structural response of device components. This generally presents itself as an improper deflection of components under load or improper mass distribution of components.

Structural similitude is the reproduction of the response of the structure to an external load, for example, hydroelasticity. The non-dimensional Cauchy number ($\rho U^2/E$) relates the influence of the flow inertial forces to the structural elastic forces. Here, the stiffness and stresses induced in an elastic structure through the interactions with the environment are desired to be reproduced. Clearly, for "rigid" structures, the elastic nature of the device response may not be dominant. However, introducing compliance into a small-scale model of a rigid non-compliant device or sub-component can bias small-scale performance testing.

Structural errors will often be introduced by improperly selecting the suitable material to manufacture the model components (due to cost or material limitations). Quantifying this error source can also be complicated, and in most cases, experience and sound engineering judgement may be necessary to identify bounds on this error source. The test model function and possible sensitivities to component deflection or improper mass distribution must be carefully evaluated before testing to assess the possible impact on measured results. If the device performance is sensitive to these factors, it is recommended that the test be designed to carefully monitor and quantify model structural response during testing.

Mass loading discrepancies may be addressed by adding mass using heavy materials etc., to areas of the model. A sensitivity study or test may be necessary to assess the impact of mass loading uncertainty (magnitude of the added mass and location of the added mass on the model).

3.3.3 Functional

Functional model errors occur in scaling sub-component elements where the function of the model element is different from that in the full-scale device. Bearings and seals are common sources of this type of error. Model scale bearings and seals may produce more friction or resistance to motion impacting load measurement under flow. These errors can often be accounted for by performing standard tare/zeroing tests to quantify any added friction in the system.

The model scale PTO can also be another source of functional error. Model scales PTOs are often not representative of the full-scale device PTO. As a result, the model scale PTO function may bias the model scale tests providing an added error in scaling up model scale results to full-scale prediction of performance. If the device performance is strongly coupled to the PTO function, then model scale testing must carefully assess any impact of PTO modelling on overall device performance.

Depending on the model scale, it may not be possible to add a generator to the PTO as a device load and a fluid, mechanical or electromagnetic load is used. These devices operate as a break or frictional load on a moving component, such as a drive shaft in a rotating turbine. A common source of error in these types of loads is due to the load characteristics (friction on the moving component) changing with ambient or environmental temperature during testing. This can result in changes in shaft torque, for example, not related to power extraction but in changes in the PTO function.

It is recommended that critical sub-components, such as the PTO, be tested in standalone configurations to carefully quantify sub-component performance as a function of the operating environment (load, temperature, and motion). If sub-component testing identifies sensitivities to environmental or test parameters, the device test plan should be designed to monitor and quantify critical parameters such as PTO temperature. Sub-component testing can quantify the errors in sub-component operation, which can then be propagated into the overall device performance uncertainty.

Cavitation may occur under specific operating/deployment scenarios (high tip-speed ratios, high current speeds, and low deployment depths) in some large-scale devices. As a result, it may be desirable to perform model scale cavitation assessment studies for determining blade tip vortex onset or cavitation breakdown conditions for a defined blade design. Model tests designed to assess cavitation potential should be conducted following the ITTC Procedures (7.5-02-03-03.1 “Model – Scale Cavitation Test,” 7.5-02-03-03.2 “Description of Cavitation Appearances,” and 7.5-02-03-03.6 “Podded Propulsor Model – Scale Cavitation Test”).

3.4 Standard Sources of Uncertainty

An uncertainty analysis of any current turbine model test must include standard sources of error associated with:

A) Instrumentation – accuracy, resolution, calibration error, user error
B) Sampling errors – digitisation errors
C) Statistical errors
D) Test procedure errors – hysteresis or bias errors introduced due to how the test is run. Are the test parameters (flow velocity, TSR, submergence, etc.) varied in a random fashion to avoid hysteresis in the test results?

The cited ITTC procedures and guidelines provide summaries of these standard error sources, how to quantify them and examples for reference.

4. FACILITY

Facility uncertainties are associated with facility operation, facility flow characteristics, model installation, and the relative scaling of model size to facility size with relevance to the impact of flow blockage on model or facility function. The following sub-sections provide a summary of these error sources. These errors can be easily quantified through careful measurements and should be propagated into the total test uncertainty as a component of specific parameter uncertainty. For example, if the tunnel velocity has a 2% spatial variation across the inflow profile of the device, this 2% variation should be propagated into the test uncertainty as an uncertainty component in the measured velocity used to assess turbine performance coefficients such as power, torque or thrust/drag. This uncertainty component is in addition to the other contributing components, such as instrument error in measuring the velocity and other bias and precision errors associated with the velocity measurement. The total velocity uncertainty would be represented by the root sum square of all the contributing components following standard uncertainty procedures outlined in the referenced ITTC guides and procedures and Coleman and Steele (1999).

4.1 Flow Quality

The power extraction capacity of a current turbine device is proportional to the cube of inflow velocity the device is exposed to. As a result, the performance of most devices will be strongly dependent on the character of the flow field the device is exposed to. Spatial and temporal flow non-uniformities in the facility can generate significant bias errors between measured and predicted performance if the predicted performance does not account for these flow non-uniformities. In general, the facility flow quality should be carefully measured and documented in any current turbine test program. This should include inflow velocity profiles to quantify spatial uniformity or gradients in the inflow to the device, axial flow profiles to assess flow direction gradients, mean flow steadiness (short-term and long-term stability in maintaining flow velocity), fluid properties such as temperature and pressure stability of the test duration, uniformity of flow properties in the test section, flow direction relative to the test section coordinate system and flow turbulence.

4.2 Bias due to setup

Uncertainties occurring from the setup will depend on how accurately the test conditions can be set and maintained throughout the test. Average flow speed, velocity profile, free-stream turbulence intensity, water level, wave conditions, fluid temperature, atmospheric pressure and others are essential parameters that must be analysed regarding their variation during the experiment and the whole experimental campaign. The test section's geometric topology and the instrumentation's relative positions used to assess test conditions are other sources of error. These questions should be addressed in any test campaign.

More details on the effect of transient flow phenomena on the performance of turbines can be found in Jesus Henriques et al. (2014), Ahmed et al. (2017), and Scarlett et al. (2019).

4.3 Controllability and Repeatability

The accuracy with which a facility can set, control and maintain a test condition, such as velocity, pressure and temperature, must be taken into account when assessing sources of error in a test. These type of error sources can be accounted for by propagating them into the appropriate variable total uncertainty (velocity, pressure or temperature) before propagating that variable uncertainty into the total uncertainty of the quantity being calculated. For example, uncertainties must be propagated for the the computation of the power coefficient

$$C_P = \frac{P}{\frac{1}{2}\rho U^3 A} \tag{1}$$

and the thrust coefficient

$$C_T = \frac{T}{\frac{1}{2}\rho U^2 A} \tag{2}$$

where, P is the measured or estimated power from other measured quantities, T is the measured shaft thrust, A is the turbine area defined by πR^2, or the maximum area of the duct, and U is the incoming flow velocity. Considering the power, the uncertainty in C_P is obtained by propagating the total uncertainties of the variables P, V and A using standard error propagation methodologies outlined in the citations referenced in this guideline, Coleman and Steele (1999) and ITTC Procedure 7.5-02-01-01. Errors in controlling the facility velocity would be appropriately included in the total velocity uncertainty before propagating the velocity uncertainty into the uncertainty in C_P.

Similar to controllability, a facility's ability or lack thereof to repeat test conditions can introduce errors in the measurements. Repeat testing can also be used to assess a model's ability to respond to a set condition in a repeatable fashion. This can be used to assess hysteresis, model or component wear, and precision errors associated with the test. In model tests requiring long test duration or the need to repeat test conditions over multiple days, the error in a facility's ability to repeat test conditions may need to be accounted for in the total uncertainty.

These errors in controllability and repeatability can be reduced if careful measurements of the test conditions are performed and synchronised with model measurements throughout the test. Data post-processing can then be performed using actual test conditions accounting for variability in control or test repeats. This methodology for reducing/accounting for facility control uncertainty will often only work if the model response being measured relative to the variable in question is well-behaved.

4.4 Installation

Model installation can introduce a source of error if the model function or performance depends on model orientation relative to the incoming flow. In such situations, careful alignment of the model relative to facility references aligned with the flow direction is recommended and should be carefully measured and quantified. Tests involving model tear-out and re-installation under repeat test conditions should be performed to quantify variability in model response due to installation. Uncertainty in model installation can then be assessed by quantifying the standard deviation in the measured model response variables over the number of repeat installation tests and applying standard student–T analyses to estimate 95% confidence uncertainty ranges.

Instrumentation installation can also introduce errors if the accurate response of the instrument in question is sensitive to alignment with the flow direction or the model. Velocity and force sensors may be sensitive to alignment relative to the incoming flow or to the model. In addition, the installation of model components such as bearings and seals can also introduce bias errors into measurements of shaft thrust and torque. It must be carefully assessed through zeroing or tare-type tests where friction introduced by these components or their misalignment with the model are quantified. This is particularly important in tests where the product of shaft torque and rpm determines the measured power.

4.5 Blockage

Flow blockage errors occur when a device, designed to operate in an open environment, is tested in a closed environment such as a water/wind tunnel, tow tank or channel and the walls or free surface of the facility constrain or alter the flow streamlines entering and exiting the device. This will often increase power production, and the amount of increase will be a function of the percentage of blockage and the flow velocity. Similar to propulsion tests, current turbine testing in a confined facility should be performed following ITTC Procedure 7.5-02-03-02.1, “Propulsor Open Water Test” and ITTC procedures and guidelines relevant to the uncertainty in powering and resistance testing (7.5-02-03-01.2, 7.5-02-02-02 and 7.5-020-05-03.3).

Well-established techniques exist to evaluate the effects of blockage on marine vehicles and structures, and hence to correct the measured data. Corrections are typically based on the ratio between the cross-section area of the model and the cross-section area of the tank. This ratio should be reduced as far as possible in order to minimise blockage effects, and in the case of energy conversion devices, to minimise the effect on device performance. Whelan *et al.* (2009) present blockage and free-surface corrections for horizontal axis devices and propose an approach to correct results in the presence of blockage in conjunction with a free surface. Ross (2010) describes a study on wind tunnel blockage corrections applied to vertical axis devices. Special consideration should be given if non-axial flow conditions, common in current turbines, are to be considered (see Bahaj *et al.* (2007)).

Nevertheless, experiments with rotors or arrays of turbines under intense blockage are becoming more common as the efficiency of farms with turbines in close proximity is being considered. See Nishino and Willden (2012), Adcock et al. (2021) and McNaughton et al. (2022) for more details on turbines operating in high blockage. Uncertainty originating on the relative position of the rotors as well as on their interference with the surrounding boundaries (free surface, walls, bottom, other rotors, support structures, etc.) must be considered and properly propagated.

4.6 Supporting Structure and Instrumentation

In a number of tests, models are installed with one or more auxiliary structures to enhance the stability of the model or to attach instrumentation effectively. Corrections shall be considered. Li and Calisal (2010) presented an analytical way to quantify the arm connection errors for vertical axis turbines and their supporting structures.

Due to the controllability of the carriage speed in most towing tanks, there are always minor vibrations reaching the turbine, especially when the speed is beyond a specific value, usually, 5 m/s for a large tank. However, to meet the correct Reynolds number, one usually must run the test fast. One solution would be to use a larger model at slower towing speeds to reduce the vibration bias. However, care must be taken to consider the trade between test scale, facility blockage, acceptable loads, dynamometer capacity and structural deformations.

Testing can become further complicated if one considers testing turbines in an array configuration. Support structures and mounting system interaction may induce further vibrations. Pintar and Kolios (2013) suggested an alternative surface towed methodology by considering scaling issues and using computational fluid dynamics to configure and size a tidal turbine array test rig in order to reduce interactions.

5. APPLICATION OF UNCERTAINTY ANALYSIS TO A HORIZONTAL AXIS CURRENT TURBINE MODEL TEST

5.1 Test Data

An 800 mm diameter horizontal axis current turbine was mounted in a water tunnel for a representative example. The rotor thrust (T) and torque (Q) are assumed to be measured using a strain-gauged load cell mounted in the hub.

5.2 Precision Limits (Type A Uncertainty)

Type A uncertainties are the evaluation of uncertainty by the statistical analysis of a series of observations (JCGM (2008) and Taylor *et al.* (1993)). This type of uncertainty is as also commonly known as the precession of the test.

Repeated tests are required to help understand the precision limits, which can also include tests in different test facilities with the same test rig. For guidance and an example of assessing precision limits see section 2.3.2 of uncertainty analysis example for open water testing of propellers (ITTC Procedure 7.5-02-03-02.2).

5.3 Bias Limits (Type B Uncertainty)

Type B uncertainties are the evaluation of uncertainty by means other than statistical analysis. (JCGM (2008) and Taylor et al. (1993)). These are also commonly unknown as bias errors.

5.3.1 Geometry

The influence of many of the errors in the manufacture of current turbine blades is difficult to estimate. Only the bias error considered in this example is the rotor radius, as it directly affects the data reduction equations. For the example, the radius (R) is 0.4 m with an accuracy of

±0.1mm, which corresponds to 0.025%. This assumes the blades are rigid or have structural similarity. If this is not the case, the propagation of additional uncertainty may be required.

5.3.2 Temperature and Density

The temperature of the water in the channel was measured to be 15.2°C with a thermometer calibrated to +/-0.2°C. By applying the method outlined in the water properties ITTC Procedure 7.5-02-01-03, "Density and Viscosity of Water". The density (ρ) is 999.072 kg/m^3 and the corresponding bias (B_ρ) is 0.0306 kg/m^3.

5.3.3 Rotational Speed

Due to the controller methodology to hold rotational speed, the accuracy of the rotational speed is limited to +/- 0.25 rpm. Therefore, the corresponding rotational speed bias (B_n) is 0.00147 rpm. This is 0.3% of the measured rotational speed of 170.0 rpm.

5.3.4 Flow Speed

The flow speed in the channel was measured using the standard Pitot tube mounted on the channel bed. Based on a survey of the wake employing Laser Doppler, the reported accuracy of the tunnel speed is 1% over the swept area of the rotor. For these tests, the measured tank speed was $U = 1.70$ m/s. Using the accuracy statement and measured speed, the total bias is therefore $B_U = 0.0170$ m/s.

5.3.5 Thrust and Torque

The ITTC procedure for calibrating instrumentation ITTC Procedure 7.5-01-02-01, "Uncertainty Analysis, Instrument Calibration" was followed. The bias in torque (B_Q) was estimated to be 0.313 Nm and thrust (B_T) to be 0.425 N. For the example calculation, the measured torque was 28.69 Nm, and the thrust was 466.60 N. The bias limits, therefore, represent 0.07% and 0.09% of the torque and thrust.

For some measurement techniques, the static friction of bearings or seals can affect the thrust and torque measurements. For this case, the bias error of the datum (also known as dynamic zero) should also be estimated. This can be achieved from the error analysis of curve fits to the turbine tested in the bollard pull condition before and after a set of test series.

5.3.6 Power

The power (P) is estimated from the rotational speed and torque.

$$P = \omega Q = 2\pi n Q \tag{3}$$

Assuming the errors in the components are not correlated, the bias in power (B_P) can be calculated from:

$$(B_P)^2 = (2\pi Q\, B_n)^2 + \left(2\pi n\, B_Q\right)^2 \tag{4}$$

For this example, the representative power and associated estimation of total bias is

P_D= 510.72 W and B_P = 5.764 W.

If the measurement methodology of rotational speed was correlated to the torque measurement, then the above simplification is not adequate, and a more detailed analysis would be required.

5.3.7 Total Tip Speed Ratio Bias

The combined bias for the tip speed (B_λ) is a combination of bias of the radius, rotational speed and tunnel speed as detailed in the equation below.

$$(B_\lambda)^2 = \left(\frac{\partial\lambda}{\partial R}B_R\right)^2 + \left(\frac{\partial\lambda}{\partial n}B_n\right)^2 + \left(\frac{\partial\lambda}{\partial U}B_U\right)^2 \tag{5}$$

where, the derivatives are:

$$\frac{\partial \lambda}{\partial n} = \frac{2\pi R}{U}$$

$$\frac{\partial \lambda}{\partial U} = 2\pi R n\left(\frac{-1}{U^2}\right)$$

The combined bias in tip speed ratio

$B_\lambda = ((1.478 \cdot 0.0002)^2 + (10.47 \cdot 0.00833)^2 + (-2.464 \cdot 0.01700)^2)^{0.5} = 0.0437$.

This corresponds to 1.0% of the tip speed ratio of 4.188.

5.3.8 Total Power Coefficient Bias

The combined bias for the power coefficient (B_{C_P}) is a combination of bias of the radius, tunnel speed, density and power as detailed in the equation below.

$$\left(B_{C_P}\right)^2 = \left(\frac{\partial C_P}{\partial R} B_R\right)^2 + \left(\frac{\partial C_P}{\partial U} B_U\right)^2 + + \left(\frac{\partial C_P}{\partial \rho} B_\rho\right)^2 + \left(\frac{\partial C_P}{\partial P} B_P\right)^2 \quad (6)$$

where the derivatives are:

$$\frac{\partial C_P}{\partial R} = \frac{P}{0.5\rho U^3 \pi}\left(\frac{-2}{R^3}\right)$$

$$\frac{\partial C_P}{\partial U} = \frac{P}{0.5\rho \pi R^2}\left(\frac{-3}{U^4}\right)$$

$$\frac{\partial C_P}{\partial \rho} = \frac{P}{0.5\rho U^3 \pi R^2}\left(\frac{-1}{\rho^2}\right)$$

$$\frac{\partial C_P}{\partial P} = \frac{1}{0.5\rho U^3 \pi R^2}$$

The combined bias limit for the turbine power coefficient is

$B_{C_P} = ((-2.070 \cdot 0.0002)^2 + (-0.731 \cdot 0.0170)^2 + (-0.00041 \cdot 0.0306)^2 + (5.764 \cdot 0.0133)^2)^{0.5} = 0.0133$.

This corresponds to 3.2% of the calculated C_P of 0.414.

5.3.9 Total Thrust Coefficient Bias

As for the thrust, the total bias of the thrust coefficient (B_{C_T}) is detailed in the equation below.

$$\left(B_{C_T}\right)^2 = \left(\frac{\partial C_T}{\partial R} B_R\right)^2 + \left(\frac{\partial C_T}{\partial U} B_U\right)^2 + + \left(\frac{\partial C_T}{\partial \rho} B_\rho\right)^2 + \left(\frac{\partial C_T}{\partial T} B_T\right)^2 \quad (7)$$

where the derivatives are:

$$\frac{\partial C_T}{\partial R} = \frac{T}{0.5\rho U^2 \pi}\left(\frac{-2}{R^3}\right)$$

$$\frac{\partial C_T}{\partial U} = \frac{T}{0.5\rho \pi R^2}\left(\frac{-2}{U^3}\right)$$

$$\frac{\partial C_T}{\partial \rho} = \frac{T}{0.5\rho U^2 \pi R^2}\left(\frac{-1}{\rho^2}\right)$$

$$\frac{\partial C_T}{\partial T} = \frac{1}{0.5\rho U^2 \pi R^2}$$

The combined bias for the power coefficient is, therefore

$B_{C_T} = ((-3.125 \cdot 0.0002)^2 + (-0.757 \cdot 0.0170)^2 + (-0.0006 \cdot 0.0306)^2 + (0.001378 \cdot 0.4259)^2)^{0.5} = 0.0129$.

This corresponds to 2.0% of the calculated C_T of 0.643.

5.4 Total Uncertainty

The total uncertainty is a combination of both the known Type A and Type B uncertainties, (the bias limits (B) and the precision limits (P)). These are combined for the tip-speed ratio and power and thrust coefficients, as detailed below.

$$\begin{gathered}(U_\lambda)^2 = (B_\lambda)^2 + (P_\lambda)^2 \\ (U_{C_P})^2 = (B_{C_P})^2 + (P_{C_P})^2 \\ (U_{C_T})^2 = (B_{C_T})^2 + (P_{C_T})^2\end{gathered} \tag{8}$$

6. PARAMETERS, SYMBOLS

6.1 List of Symbols

A	turbine swept area,	m^2
B_i	bias uncertainty in variable i	
C_P	power coefficient	
C_T	Thrust coefficient	
E	bulk modulus of Elasticity,	Pa
i	variable	
n	rotational rate,	Hz
P	Shaft Power,	W
P_i	precision uncertainty in variable i	
Q	shaft torque,	N·m
R	turbine radius,	m
T	shaft thrust,	N
U	upstream velocity,	m/s
u	flow velocity,	m/s
λ	tip speed ratio,	
ρ	density,	$\mathrm{kg/m}^3$
ω	rotational velocity,	rad/s

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