ITTC Quality System Manual
Recommended Procedures and Guidelines

Procedure

Frequency of Random Events

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Frequency of Random Events

1. PURPOSE OF PROCEDURE

Detailed guidance on the estimation of frequency of random events is provided. These events are stability failures observed in numerical time-domain simulation of ship motions in irregular waves carried out for direct stability assessment (DSA). The DSA is a part of the second-generation IMO intact stability criteria, described in the Interim Guidelines, MSC.1/Circ. 1627, IMO (2020).

The DSA procedures are intended is to employ latest technology in time-domain numerical simulation of ship motion. The calculation frequency of random events is aimed on statistical characterization of stability failures it is identified as "direct counting" and described in section 3.5.4 of MSC.1/Circ.1627 (IMO 2020), section 3.5.4 of the Explanatory Notes, MSC.1/Circ.1652 (IMO 2023), while more details are available from sections 3.3 through 3.5 of Appendix 4 to the Explanatory Notes. Theoretical background of these methods and assessment of their uncertainty is available from Wandji et al. (2024). Information on the benchmarking and testing of these method can be found in (Shigunov et al. 2022) and (Wandji et al. 2023). Calculation of frequency of random events for a given sea state, speed and heading is described in the present procedure.

2. INTRODUCTION

2.1 Formulation of the Problem

Stability failure in the DSA framework is defined in paragraph 3.2.1 of Interim Guidelines (MSC.1/Circ.1627, IMO 2020) include exceedance of 40 degrees roll or 9.81 m/s^2 lateral acceleration at the highest location where passengers and crew may be present.

Time-domain numerical simulations generate a set of records of roll angle and the lateral accelerations. Rate of the observed stability failures need to be estimated from this set of records. A confidence interval should be constructed to characterize statistical uncertainty of the rate estimate.

2.2 Scope of the Procedure

The present procedure is limited to the statistical processing of a number of observed stability failures. Requirements for numerical simulation tools are described in the sections 3.3 and 3.4 of the Interim Guidelines (MSC.1/Circ.1627, IMO 2020) and are outside of the scope of the present recommended procedure. Requirements for setting and running the simulations, described in the sections 3.5.2 and 3.5.4 of IMO (2020) are also outside of the scope of the present recommended procedure.

Finally, the present procedure is only relevant for full probabilistic assessment (section 3.5.3.2 of the referred Interim Guidelines) and assessment in design situation using probabilistic criteria (section 3.5.3.3 of the referred Interim Guidelines).

2.3 Theoretical Background

Stability failures are modelled as a Poisson flow of events. Three assumptions are associated with the Poisson flow of events: the events are independent, probability of an event at a particular instant of time is infinitely small and only one event can occur at a particular instant of time.

A probability of at least one event occurring during a given time T is expressed as:

$$P(N > 0) = 1 - \exp(-rT), \tag{1}$$

where N is a number of random events and r is the rate of events.

Time before the first event occur or time between the two consequent events is a random variable following exponential distribution with probability density function (PDF):

$$\mathrm{PDF}(T) = r \cdot \exp(-rT), \tag{2}$$

The mean value μ_T of the time T is expressed through the rate of events as:

$$\mu_T = \frac{1}{r}. \tag{3}$$

More information on the probabilistic framework is available from sections 2.1 and 2.2 of SDC 8/INF.2, IMO (2021) — the document describing the physical background and mathematical models for stability failures of the second generation intact stability criteria. The rate of events is estimated as:

$$\hat{r} = \frac{N}{T}. \tag{4}$$

The “hat” symbol $\hat{\square}$ means “estimate” rather than a theoretical or “true” value.

The estimate is a random value, and its variability or statistical uncertainty is characterized with a confidence interval. The probability that unknown “true” value belongs to the confidence interval is defined as a confidence probability P_β. The boundaries of the two-sided confidence interval $\hat{r}_{low}$ and $\hat{r}_{up}$ for the estimate $\hat{r}$ are expressed as:

$$\hat{r}_{low} = Q_r\left(\frac{1-P_\beta}{2}\right)$$
$$\hat{r}_{up} = Q_r\left(\frac{1-P_\beta}{2}\right) \tag{5}$$

where Q_r is a quantile of the distribution of the estimate $\hat{r}$, i.e. an inverse of the cumulative distribution function (CDF) of $\hat{r}$.

3. DATA REQUIREMENTS

The sample data set (ensemble) consists of several independent records. Records can be of different duration for the method based on exponential distribution (Section 4 of this Procedure) and the method based on binomial distribution (Section 6 of this Procedure). The records have to be of the same duration for the method based on the probability estimation (Section 5 of this Procedure).

A nested array (array having other arrays as elements) is a convenient way to describe this type of data.

$$X = \{X_j;\ j = 1, \dots, N_r\}$$
$$X_j = \{x_{j,i};\ i = 1, \dots, N_j\}, \tag{6}$$

where X is used to identify the entire sample, the index j, identifies a record and the index i a point within a record. N_r is the number of records available in the sample, and N_j is the number of data points in the jth record. X_j identifies the jth record.

The duration of the simulation is limited by the self-repeating effect (Recommended Procedure 7.5-02-07-09, ITTC 2024). Some of the records may not contain any stability failures.

Each record should be trimmed from the beginning to exclude initial transient or ramp-up time (Paragraph 3.5.4.3 of the Explanatory Notes MSC.1/Circ.1652, IMO 2023).

A time instant of occurrence of the event *te* may be defined as the next time instant after the exceedance occurred. Alternatively, *te* may be defined as the time instance of crossing and computed with linear interpolation:

$$te = \frac{a - x_k}{x_{k+1} - x_k}\Delta t + t_k. \quad (7)$$

where a is the level established by paragraph 3.2.1 of MSC.1/Circ.1627 (IMO 2020), Δt is the time increment and k index of the time instant of the time instance, preceding to the event.

4. METHOD BASED ON EXPONENTIAL DISTRIBUTION

4.1 General

The method is based on running the simulation until the first event occur and report the value of time-before -event. A mean value of the time-before-event is used for estimate of the rate of event, equation (3). The method is introduced in paragraph 3.5.4.4.1 of the Interim Guidelines (MSC.1/Circ.1627, IMO 2020), whereas a procedure briefly described in paragraph 3.5.4.4.1 of the Explanatory Notes (MSC.1/Circ.1652, IMO 2023) with background information available from the section 3.3 of the Appendix 4 to the cited Explanatory Notes.

A stability failure event may not be observed within the record as the duration of a record is limited by the self-repetition effect. If the simulation continues to the next record in the ensemble, the time of previous record is added, until the failure is observed.

If the simulation is stopped, this situation represents a case of partially known or missing data. Then the right censoring (i.e. data point is beyond the duration of the record) is applied (e.g. section 8.1.3.1 of NIST/SEMATECH, 2012), where the event has been assumed to be observed at the end of the record.

The method is convenient when the evaluation of dynamic stability in waves in the primary objective of the simulation. The method is also convenient to apply "on the fly", i.e. until needed number of failures has been observed. The rate estimate $\hat{r}$ is assumed to follow chi-squared distribution degrees of freedom equal to twice a number of observed events. Background and justification of the method has been described by Shigunov (2019).

4.2 Estimation of Frequency and its Uncertainty

To enable right-censoring for the "on-the-fly" application, two counters for the failure event are introduced: n_i and n_i^*. For the first record of the duration T_1, the event counter n_1 is set to 1, if the failure event was observed at the end of the record and if no event occurs, $n_1 = 0$. The second counter n_1^* is set to 1, whenever the event was observed or not.

For the i-th record (i >1) of the duration T_i, counter n_i is increased by 1, if the failure event was observed and retains its previous value if no event occurs, while the second counter is set $n_i^* = n_{i-1} + 1$.

The censored event counter n_i^* differs by one from the non-censored event counter n_i^* only if the i-th record does not end up with a failure event. The "on-the-fly" estimate after i-th record is computed as:

$$\hat{r}_i = \frac{n_i}{\sum_{k=1}^{i} T_k} \quad (8)$$

The "on-the-fly" estimate after i-th record with censoring is computed as:

$$\hat{r}_i^* = \frac{n_i^*}{\sum_{k=1}^{i} T_k} \quad (9)$$

The upper boundary of the confidence interval for the rate estimate is the main output of the procedure. It is computed with the censored data:

$$\hat{r}_{Ui} = \frac{0.5\hat{r}_i^* \cdot \chi^2_{0.5(1+P_\beta),2n_i^*}}{\sum_{k=1}^{i} T_k} \qquad (10)$$

where $\chi^2_{q,k}$ is a chi-squared value computed with k degrees of freedom and corresponding to q probability, i.e. q-quantile of chi-squared distribution with k degrees of freedom. The lower boundary serves for general information and is computed without censoring:

$$\hat{r}_{Li} = \frac{0.5\hat{r}_i \cdot \chi^2_{0.5(1-P_\beta),2n_i}}{\sum_{k=1}^{i} T_k} \qquad (11)$$

5. METHOD BASED ON PROBABILITY ESTIMATION

5.1 General

The probability of occurrence is estimated for at least one event during a record, and then the estimate of rate is deduced from equation (1). All the records have to be of the same duration.

The method is introduced in paragraph 3.5.4.4.2 of the Interim Guidelines (MSC.1/Circ.1627, IMO 2020), whereas a procedure briefly described in paragraph 3.5.4.4.2 of the Explanatory Notes (MSC.1/Circ.1652, IMO 2023) with background information available from the section 3.4 of the Appendix 4 to the Explanatory Notes, IMO (2023).

The method is convenient when the evaluation of dynamic stability in waves is one of the objectives of the simulation. Generated data are meant to be used for other applications as well. Constant duration of a record simplifies the planning.

As the method is based on “at least one event” probability per record, data on the second and following event during the record will not be used. The method cannot be employed when events are observed during every single record, but special post-processing can be applied, by dividing records into pieces. The confidence interval is constructed with the F-distribution for the rate estimated with this method.

5.2 Estimation of Frequency and its Uncertainty

A single counter for events is introduced n_i. If one or more failure event have been observed in during the i-th record, n_i is set to one.

Total number of records with at least one event is calculated:

$$N = \sum_{i=1}^{N_r} n_i. \qquad (12)$$

For the case $N < Nr$, the estimate of probability of at least one failure event $\hat{P}$ is computed as:

$$\hat{P} = \frac{N}{Nr} \qquad (13)$$

The rate of failure events is estimated as:

$$\hat{r} = -\frac{1}{Tr}\ln\left(1-\hat{P}\right) \qquad (14)$$

where Tr is the duration of the record.

For the case $N = Nr$, an approximate to to reach independence τ_{ind} has to be estimated as described in subsection 3.2.3 an Figure 5 of the ITTC Recommended Procedure 7.5-02-01-08 (ITTC 2017) using autocorrelation function estimated as described in subsection 3.2.1 of the cited procedure.

Each of the original record is divided into a number of pieces of same duration Np duration of each piece Tp should not be less than the approximate to reach independence: $\tau_{ind} < Tp$.

The counting of events is repeated for the each piece. If $N = Np$, the method is not applicable as stated in paragraph 3.4.4.2 of the Explanatory Notes (MSC.1/Circ.1652, IMO 2023) and other methods, described in Section 4 or in Section 6 of ITTC (2017).

5.3 Uncertainty of the Estimate

Confidence interval is constructed from the F-distribution for the estimate of probability of at least one failure event. The F-distribution describes a fraction of two independent chi squared random variables and has two parameters: degrees of freedom of these variables. The upper boundary $\hat{P}_U$ of the confidence interval is computed as:

$$\hat{P}_U = \frac{\nu_1 F_{0.5(1+P_\beta),\nu_1,\nu_2}}{\nu_2+\nu_1 F_{0.5(1+P_\beta),\nu_1,\nu_2}} \quad (15)$$

where F_{q,ν_1,ν_2} is an F-value corresponding to q probability and computed with degrees of freedom ν_1 and ν_2, i.e. q-quantile of F-distribution. The number of degrees of freedom are computed as:

$$\nu_1 = 2(N+1);\ \ \nu_2 = 2(N_r - N) \quad (16)$$

The lower boundary $\hat{P}_L$ of the confidence interval is computed as:

$$\hat{P}_L = \frac{\nu_1 F_{0.5(1-P_\beta),\nu_1,\nu_2}}{\nu_2+\nu_1 F_{0.5(1-P_\beta),\nu_1,\nu_2}} \quad (17)$$

The number of degrees of freedom for the lower boundary are different and are computed as:

$$\nu_1 = 2N;\ \ \nu_2 = 2(Nr - N + 1) \quad (18)$$

The upper $\hat{r}_U$ and lower $\hat{r}_L$ boundary of the confidence interval for the estimate of the rate of events are computed as:

$$\hat{r}_U = -\ln\left(1-\hat{P}_U\right)/Tr \quad (19)$$

$$\hat{r}_L = -\ln\left(1-\hat{P}_L\right)/Tr \quad (20)$$

For the case $N = N_r$, if the original records were divided into pieces, the duration of piece Tp is entered in formulae (19) and (20) instead of the duration of original record Tr.

6. METHOD BASED ON BINOMIAL DISTRIBUTION

6.1 General

The estimate of rate of events is assume to follow binomial distribution in this method. The method is introduced in section 3.5 of Appendix 4 of the Explanatory notes (MSC.1/Circ.1652, IMO 2023).

Records of different duration limited are allowed only by the self-repetition effect. The method is convenient when the evaluation of dynamic stability in waves is one of the objectives of the simulation i.e. generated data are meant to be used for other applications as well.

The method can account for more than one event per record but requires that all counted events are independent. If the events are clustered i.e. observed too close to each the independence assumption may be no longer valid. To ensure independence a declustering procedure needs to be carried out.

The confidence interval is constructed from the binomial distribution for the rate estimate. Background and justification of the method has been described by Leadbetter et al. (2019).

6.2 Data

Preparation of the data requires estimation of the approximate time to reach independence, τ_{ind}, referred also as a decorrelation time in paragraph 3.5.3 of Appendix 4 of the Explanatory notes (MSC.1/Circ.1652, IMO 2023). This calculation is described in subsection 3.2.3 and Figure 5 of ITTC (2017) with autocorrelation function estimated as described in subsection 3.2.1 of the cited procedure.

6.3 Estimation of Frequency

The events that were recorded within the time difference τ_{ind} are assumed to be dependent. These events are grouped. These groups comprise a series of successive events, which are from the same record and for which the time interval between the events is less than τ_{ind}. As these groups will consist of different number of events, a nested array is a natural form for organizing the data on time of the events, *te*, for the *j*-th record:

$$\begin{aligned} C &= \{C_j;\ j = 1, \dots, N_r\} \\ C_j &= \{C_{j,i};\ i = 1, \dots, Nc_j\} \\ C_{j,k} &= \{te_{j,i,k};\ k = 1, \dots, Nec_{j,i}\}, \end{aligned} \tag{21}$$

where $Nec_{j,i}$ is the number of events in the *i*th cluster of *j*th record and Nc_j is the number of clusters in the *j*th record.

As the intervals between the clusters exceeds the time difference τ_{ind}, each cluster is considered as an independent event. The cluster duration is also a nested array and for *i*th cluster in the *j*th record is computed as:

$$Tc_{j,i} = te_{j,i,Nec_{j,i}} - te_{j,i,1} \tag{22}$$

A total number of events, to be included in the estimation equals to the number of clusters over all the records:

$$N = \sum_{j=1}^{N_r} Nc_j \tag{23}$$

Sum of the durations of the clusters needs to be subtracted from the total time as Poisson process event, since the Poisson Process is interpreted as a point process and an event must occur within a single time increment of the simulation. Usually the correction for the duration of clusters is small.

$$T_t = \sum_{j=1}^{Nr} \left(Tr_j - \sum_{i=1}^{Nc_j} Tc_{j,i} \right) \tag{24}$$

The rate of events is estimated as:

$$\hat{r} = \frac{N}{T_t}. \tag{25}$$

6.4 Uncertainty of the Estimate

The confidence interval is constructed with the binomial distribution. The binomial distribution has two parameters: total number of time increments (trials) N_t and a probability of an event occurring at any time increment (trial) *p*. The former is computed as:

$$N_t = \text{round}\left(\frac{T_t}{\Delta t}\right). \tag{26}$$

Rounding may be required if time of events are computed with linear interpolation (7). The probability of an event occurring at any time increment is estimated as:

$$\hat{p} = \frac{N}{N_t} \tag{27}$$

Boundaries of the confidence interval are computed with quantile if the binomial distribution Q_B:

$$\hat{r}_{U,L} = \frac{1}{T_t} Q_B\left(0.5(1 \pm P_\beta), N_t, \hat{p}\right) \tag{25}$$

For the cases when number of events exceed 25, normal approximation can replace the binomial distribution. The variance of this normal approximation is defined as:

$$\hat{V}_N = N_t \hat{p}(1 - \hat{p}) \approx N \tag{26}$$

as the value of $\hat{p}$ is small

$$\hat{r}_{U,L} = \hat{r} \pm \frac{\sqrt{\hat{V}_N} \cdot Q_N\left(0.5(1 + P_\beta)\right)}{T_t} \tag{27}$$

7. LIST OF SYMBOLS

F_{q,ν_1,ν_2}	Quantile q of F-distribution with the degrees of freedom ν_1 and ν_2

N	Number of events
Nc	Number of clusters
Nec	Number of events in a cluster
Np	Number of "pieces" or fragments of a record
N_r	Number of records
N_t	Number of time increments with correction for duration of clusters
n_i	Counter of events
n_i^*	Counter of events with censoring
P	Probability
P_β	Confidence probability
PDF	Probability density function
Q_r	Quantile of distribution of rate estimate
r	Rate of events, [s^{-1}]
T	Time of exposure, [s]
Tc	Duration of a cluster, [s]
Tp	Duration of a "piece" or fragment of a record, [s]
Tr	Duration of a record, [s]
T_k	Time before k-th event
T_t	Total time of exposure corrected for duration of clusters, s
te	Time instant of occurrence of an event, [s]
μ_T	Mean value of a time before event, [s]
τ_{ind}	Time to reach independence, [s]
$\chi^2_{q,k}$	Quantile q of chi-square distribution with k degrees of freedom
$\hat{\square}$	Estimate ("hat" above a symbol)
$\square_U$	Upper boundary of confidence interval
$\square_L$	Lower boundary of confidence interval

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Appendix A. CALCULATION EXAMPLE

Example for the procedure uses ITTC-A1 ship (Umeda et al., 2000), whose principal particulars are summarized in Table A1. This configuration was used in the ITTC benchmarking (ITTC, 2005) and SAFEDOR project (e.g. Spanos and Papanikolaou 2009). Roll decay data were available from the latter reference. The lines are shown in Figure A1.

Fast volume-based simulation tool (Weems et al. 2023) was used to generate sample data. Simulation included 3 degrees of freedom: heave-roll-pitch. While these three degrees of freedom may be insufficient for direct assessment for all modes of failure under the Second Generation IMO Stability Criteria per MSC.1/Circ. 1627, this simplified simulation is capable of producing the data to serve as an example.

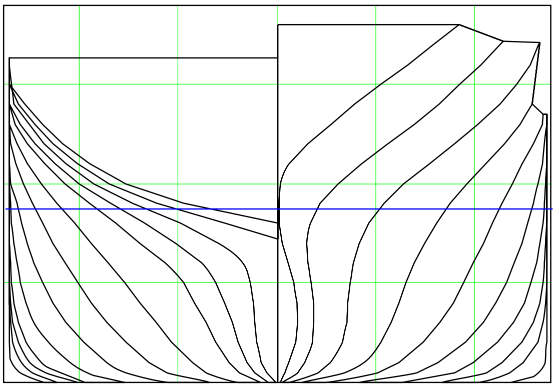

Figure A1: Lines of ITTC-A1 ship

Table A1: Principal input data

Length BP, m	150
Breadth, m	27.2
Draft amidships, m	8.5
KG , m	10.24
GM ,m	1.38
CB	0.667
CM	0.959
CW	0.786

A body-nonlinear formulation was applied for hydrostatic and Froude-Krylov forces. Diffraction and radiation for heave and pitch and diffraction for roll was approximated from potential flow simulation tool LAMP (Large Amplitude Motion Program), while added mass and damping for roll was extracted from roll decay test of (Spanos and Papanikolaou 2009). Details are in Weems et al (2023).

Wave environment was represented by long-crested irregular waves generated with Bretschneider (1959) spectrum recommended by 1978 ITTC Performance Prediction Method, ITTC (2021). The significant wave height was 7.5 m and modal period 14 s. The spectrum was discretized with 240 frequencies from 0.2 to 0.8 1/s. The time step was 0.5 s, with the ramping time of 10 s. Calculations were done for forward speed of 10 knots at stern quartering heading angle of 45 degrees. Duration of a record without self-repeating effect was 40 min. Totally, 100 records were produced, covering 66.4 hours.

Three capsizes were detected. For estimation of mean, standard deviation and autocorrelation function, portions of the records, where capsizings where detected, were removed above 50 degrees, to ensure stationarity of the roll motion data. As a result, 49 minutes were removed from the dataset, decreasing total time to 65.6 hours.

Ensemble averaged estimates of mean variance and standard deviation were computed as estimated as described in ITTC (2017) Recommended Procedure 7.5-02-01-08 “Single Significant Amplitude and Confidence Intervals for Stochastic Processes”. Numerical results are placed in Table A2. Autocovariance function was estimated as recommended in ITTC (2017). Figure A2 illustrates ensemble averaging of the estimates of autocovariance function.

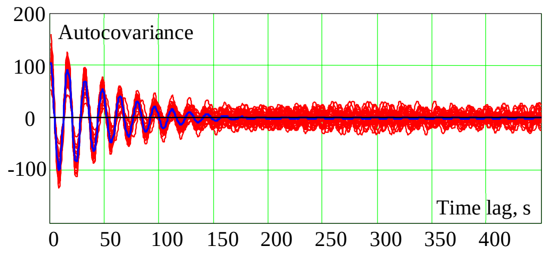

Figure A2: Estimation of the ensemble-averaged autocovariance function (blue). Record estimates are in red.

Figure A3 describes estimation of decorrelation time, following ITTC (2017). Autocorrelation function was computed by normalizing the estimate of autocovariance function by the variance estimate. Envelope of ensemble-averaged autocorrelation function was constructed, and its intersection with the level of significance of 5% was found to be at about 150 seconds. This value was taken as a decorrelation time.

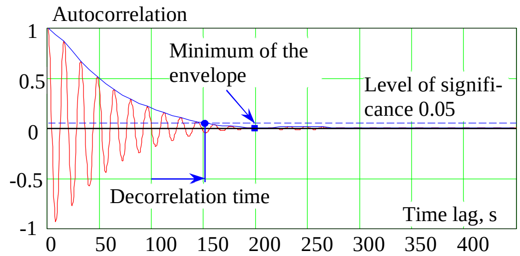

Figure A3: Estimation of decorrelation time

Table A2: Roll motion ensemble characteristics and estimates

Duration of record, min	40
Total number of records	100
Total time, h	66.4
Number of capsizing	3
Total time after capsizings, min	49
Total time of stationary data, h	65.6
Estimate of mean value, deg.	-0.837
Estimate of standard deviation, deg.	10.15
Estimate of decorrelation time, s	150.1

Search for exceedances of 40 degrees was performed for both starboard and port sides. Time instant of exceedance was computed with linear interpolation. The exceedance data are summarized in Table A3, the record 19, included in Table A3 is illustrated in Figure A4.

Table A2: Roll motion ensemble characteristics and estimates

#	Record	Level, deg	Time, s	Capsizing time if any, s
1	14	-40	2238.1	-
2	19	-40	812.6	-
3	23	-40	2219.5	-
4	26	-40	236.9	-
5	26	40	245.6	261.0
6	62	-40	1432.5	-
7	65	-40	1725.2	-
8	74	-40	1944.3	-
9	74	-40	1952.2	1966.0
10	94	-40	1991.0	2002.5

Intermediate and final results of estimation of rate of exceedance of 40 degrees angle is placed in Table A3. The methods based on probability estimation and binomial distribution have detected 8 independent events, while the method based on exponential distribution uses 9 events due to censoring the last record that did not have any exceedances. Final results are plotted in Figure A5. All three methods have shown almost identical results in the considered example

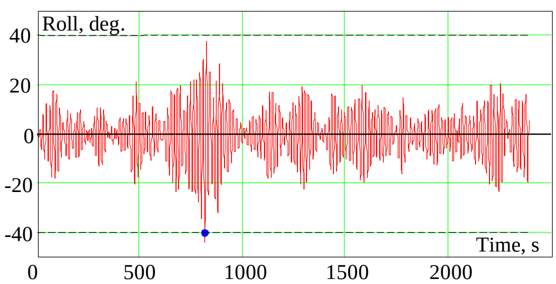

Figure A4: Record #19 with exceedance at 812 s

Table A3: Intermediate and final results for estimation of frequency of random events

Method based on exponential distribution	
Total time of exposure, s	232480
Number of exceedances for estimate	9
Estimate of rate, s^{-1}	$3.871 \cdot 10^{-5}$
Upper boundary for rate, s^{-1}	$6.78 \cdot 10^{-5}$
Lower boundary for rate, s^{-1}	$1.77 \cdot 10^{-5}$
Method based on probability estimation	
Time of exposure, s	2390
Number of records with at least one event	8
Estimate of probability	0.08
Upper boundary for probability	0.152
Lower boundary for probability	0.035
Estimate of rate, s^{-1}	$3.49 \cdot 10^{-5}$
Upper boundary for rate, s^{-1}	$6.87 \cdot 10^{-5}$
Lower boundary for rate, s^{-1}	$1.50 \cdot 10^{-5}$
Method based on binomial distribution	
Total time of exposure, s	236401
Number of independent events	8
Variance of the number of events	8
Lower binomial quantile	3
Upper binomial quantile	14
Estimate of rate, s^{-1}	$3.39 \cdot 10^{-5}$
Upper boundary for rate, s^{-1}	$5.74 \cdot 10^{-5}$
Lower boundary for rate, s^{-1}	$1.04 \cdot 10^{-5}$

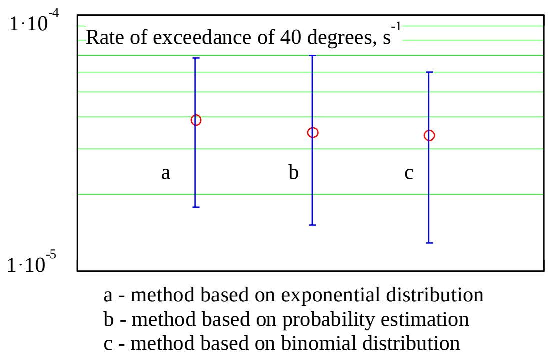

Figure A5: Results of estimation of frequency random events – direct counting