ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Determination of a Type A Uncertainty Estimate of a Mean Value from a Single Time Series Measurement

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Disclaimer

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Determination of a Type A Uncertainty Estimate of a Mean Value from a Single Time Series Measurement

1. PURPOSE OF PROCEDURE

Single time histories such as those obtained from measurements in a towing tank, wind tunnel, or full-scale ship trial are often subjected to low frequency random disturbances caused by long transient responses to start-up conditions (e.g. towing carriage acceleration profile), low damping, recirculation effects and a host of external factors such as varying environmental conditions. Such disturbances cause a random low frequency variation in the mean value of the measured time history.

Accurate estimation of the mean value can be problematic in cases where the low frequency random variation in the mean value is large compared to the estimated mean value. The 'traditional' approach to obtaining a 'good' estimate for the mean value of such 'non-stationary' signals is to increase the length of the measurement time, requiring a very long towing tank/test region. The assumption with this approach being that the non-stationary behaviour diminishes with time, which may not always be the case. The more common practice is to carry out multiple repeat tests and combine the results in order to obtain a mean value within acceptable limits. However, the requirement to perform multiple repeat tests is often at odds with the commercial realities of limited time and test budgets.

Methods have been suggested in Lin et al. (1990) and in Molloy (2010), which process a single time series and obtain an estimate for the level of uncertainty. These and other traditional methods require subjective choices, such as the number and length of the subdivision of the time series, to be made. As such, these methods are difficult to develop as a procedure.

More recently, Brouwer et al. (2013), (2015a) proposed two objective procedures for estimating the uncertainty from a single time series measurement based on autocovariance and segment method, the former being favoured by the authors for its rapid convergence. Both procedures are based on JCGM (2008) and evaluate the random uncertainty of mean with statistical techniques. JCGM (2008) classifies this method as Type A evaluation.

A Transient Scanning Technique, TST, was proposed by Brouwer et al. (2015b). This technique determines whether a signal approaches a stationary state or not and also provides information to allow non-stationary behaviour (typically from start-up and end effects) to be removed from the signal so that the uncertainty in the mean value can be estimated with the methods in Brouwer et al. (2015a), (2019) more accurately. A summary of this work comprises the majority of this document. In this revision, the earlier references of Brouwer et al. (2013) and (2015a, b) have been consolidated and updated in Brouwer et al. (2019).

The TST is a practical tool to verify whether the mean value is constant or not. Further, it locates trends (or transients) in measured signals at the beginning or at the end of time series. These transients are often too small to see in visual checks, but with the TST, these transients can be identified and, hence, removed from the signal. The residual part of the time series is stationary and can be applied for further analysis.

Section 2 explains briefly the theoretical background of the autocovariance method. In

Section 3, the TST is described. In Section 4, the technique is demonstrated by synthetic data. Section 5 provides a step-by-step guide to implementing the procedure. References are provided in Section 6.

2. DESCRIPTION OF PROCEDURE

2.1 Theoretical Background

This section provides a brief description of the theoretical background presented in detail in Brouwer et al. (2013), (2015b), and (2019).

A time series *x*(*t*) with finite length *T* is considered as a sample record of an ergodic stationary random process. The sample average m_1 or first moment as defined in Bendat and Piersol (2010) is an estimate of the true mean of the process μ_x.

$$m_1 = \frac{1}{T}\int_0^T x(t)dt \quad (1)$$

For a digital time series, the result is

$$m_1 = \bar{x} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} x_i \quad (1a)$$

where n is the number of samples. The sample variance s_x^2 is an estimate of the true variance of the process σ_x^2

$$s_x^2 = \frac{1}{T}\int_0^T (x(t) - m_1)^2 dt \quad (2)$$

while the digital result is

$$s_x^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(x_i - \bar{x})^2 \quad (2a)$$

Due to the finite length *T* of the time series, a difference will occur between the estimated mean m_1 and the true mean μ_x.

A reliable measure of the random uncertainty u_1 of mean is the standard deviation s_m of mean, which is the square root of the variance of mean. The expected value E[ ] of the variance of mean can be written as

$$s_m^2 = E[(m_1 - \mu_x)^2] \quad (3)$$

Substituting Equation (1) into Equation (3) and after some manipulation following Bendat and Piersol (2010), the variance of the mean can be written as:

$$s_m^2 = \frac{2}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) C_{xx}(\tau)d\tau \quad (4)$$

With

$$C_{xx}(\tau) = \frac{1}{T}\int_0^T x_i(t)\cdot x_i(t+\tau)dt \quad (5)$$

with

$$C_{xx}(0) = \mu_x^2$$

where:

s_m	standard deviation of the mean,
T	measurement length
τ	time difference or lag
Δt	sampling interval
$C_{xx}(\tau)$	autocovariance function

for a stationary process

The form for digital processing is

$$s_m^2 = \frac{s_x^2}{n} = +\frac{2}{n}\sum_{r=1}^{n}\left(1 - \frac{r}{n}\right) C_{xxr} \quad (4a)$$

The autocovariance can be estimated via direct computations:

$$C_{xxr} = \left(\frac{1}{n-r}\right)\sum_{i=1}^{n-r} x_i x_{i+r} \quad (5a)$$

$$r = 1, 2, 3, \cdots, n-1 \quad (5b)$$

and

$$C_{xx0} = \mu_x^2 \quad (5c)$$

Another method of computing autocovariance is an indirect, spectral method. Its estimate is based on the Wiener-Khinchine relation

$$C_{xx}(\tau) = \int_0^\infty S_{xx}(f) \cdot cos(2\pi f\tau)df \quad (6)$$

The auto spectral density function $S_{xx}(f)$ is computed from the Fourier transform $X(f)$:

$$S_{xx}(f) = \frac{2}{T}|X(f)|^2$$
$$X(f) = \int_0^T (x_i(t) - m_1)e^{-j2\pi t}dt. \quad (7)$$

Bendat and Piersol (2010) give a complete numerical procedure to compute the autocovariance with Equation (6). The latest step of their procedure is to compute the unbiased autocovariance $C_{xx,\text{unbiased}}(\tau)$ from the biased estimate $C_{xx,\text{biassed}}(\tau)$ according to

$$C_{xx,\text{unbiased}}(\tau) = \frac{T}{T-|\tau|} C_{xx,\text{biased}}(\tau) \quad (8)$$

Brouwer et al. (2013, 2019) use the biased estimator $C_{xx,\text{biased}}(\tau)$ for the autocovariance to reduce numerical instabilities in Equation (8) for large values of τ.

Consequently, the biased estimator $C_{xx,\text{biased}}(\tau)$ returns a biased estimate $s^2_{m,\text{biased}}$ of the variance of mean. Brouwer et al. (2013), (2019) show that the ratio of biased to unbiased estimate can be approximated with:

$$\frac{s^2_{m,\text{biased}}}{s^2_{m,\text{unbiased}}} = 2 \quad (9)$$

The random uncertainty u_1 of mean is the square root of the variance of mean

$$u_1 = s_m = \sqrt{s^2_m} \quad (10)$$

Combining this with Equations (4) and (9) gives a relation for the random uncertainty of mean based on the autocovariance method:

$$u_1 = \sqrt{\frac{1}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) C_{xx,\text{biased}}(\tau)d\tau} \quad (11)$$

The corresponding expanded uncertainty U_m of mean is obtained by multiplying Equation (11) with a coverage factor k_{95}

$$U_m = k_{95}u_1 \quad (12)$$

with k_{95} = 1.96 for a confidence level of 95%.

Brouwer et al. (2019) call this evaluation method the Random Uncertainty of Mean estimator (RUM).

For uncorrelated samples (or white noise signals), the autocovariance is zero, despite the zero-lag value:

$$C_{xxo} = s^2_x$$
$$C_{xxr} = 0 \;\; for\; r \neq 0 \quad (13)$$

Then, Equation (4a) becomes

$$u_1 = s_m = s_x/\sqrt{n} \quad (14)$$

However, pure white noise signals do not exist in practice. Some correlation always exists between successive samples.

The autocovariance method incorporates the correlation between the samples in a signal (or time series). Brouwer et al. (2013, 2019) have shown how to compute the expected u_1, for stationary stochastic correlated processes when T is sufficiently long. For such processes, the standard uncertainty u_1 of mean decays as the sample length T increases.

The inverse relation between u_1 and T can be verified visually in a graph with T plotted on the x-axis and u_1 on the y-axis, both with logarithmic scales. For stationary signals, the trend should form a line with a slope of minus one. If the slope differs from minus one, then the signal is non-stationary. Figure 4 is an example.

3. TRANSIENT SCANNING TECHNIQUE

By the autocovariance method, Brouwer et al. (2013), (2019) developed a second technique, called the Transient Scanning Technique, TST. The TST identifies regions in the time history, which exhibit non-stationary behaviour and allows these regions to be removed from the signal analysis in order to obtain a more accurate estimate of the mean value. A TST is constructed by calculating the cumulative u_1 as shown in Equation (11) for a range of sample lengths T. The TST can be applied in two ways.

3.1 TST-A

Starting from the beginning of the signal t_{begin} the TST select signal sections $[t_{begin}, t_{begin} + T]$. Then, or each section size T compute the autocovariance on that section, with spectral method and calculate the uncertainty of mean from Equation (11). The series of u_1 are plotted against T on logarithmic scales. TST-A is very strong in identifying end effects in the signal.

3.2 TST-B

Starting from the end of the signal $t = t_{tend}$, the TST selects signal sections $[t_{end} - T, t_{end}]$. Then for each T, compute the autocovariance $C_{xx,\text{biassed}}(\tau)$ on that section with the spectral method, and calculates the uncertainty u_1 of mean from Equation (11). The results are plotted against T on logarithmic scales. TST-B is very strong in identifying start-up effects in the signal.

4. EXAMPLE

To demonstrate the TST, a realisation is picked from an artificially created stochastic process. Its mean is zero, and it has no energy below 0.25 Hz. The realisation length is 100 s, which is long enough to be stationary. A second signal is created from the realisation, which is made non-stationary by adding a small start-up effect in the first 20 s. Both signals and the disturbance are shown in Figures 1, 2, and 3. More details can be found in Brouwer et al. (2019).

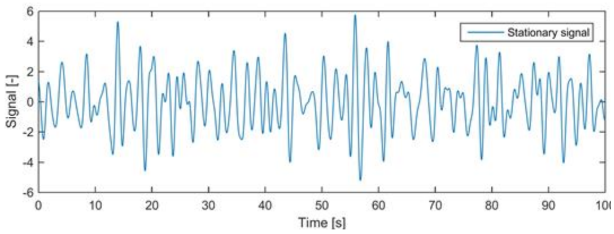

Figure 1: Realisation taken from a stochastic process from Brouwer et al. (2019)

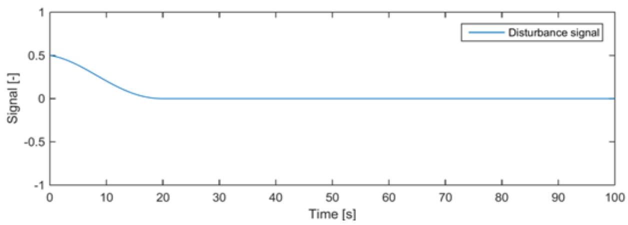

Figure 2: Start-up disturbance from Brouwer et al. (2019)

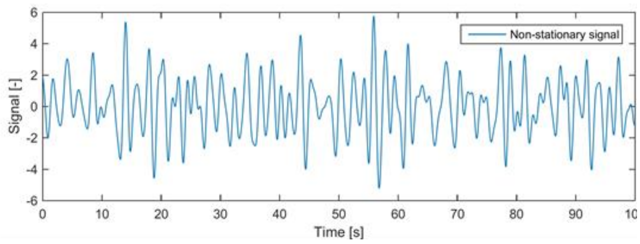

Figure 3: Realisation with added start-up disturbance from Brouwer et al. (2019)

Checking for stationarity by visual inspection of the signal in Figure 3 is difficult, since the start-up effect has much smaller amplitude than the signal. Figure 4 shows the TST results for the stationary signal and the TST for the non-stationary signal is shown in Figure 5. In order to identify the start-up effects, the uncertainty values u_1 are calculated from the end (TST-B) by the autocovariance method.

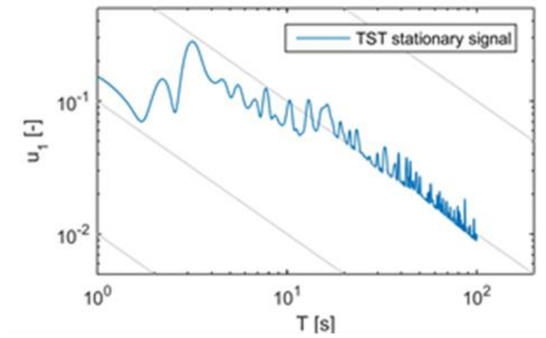

Figure 4: TST-B applied to original stationary signal from Brouwer et al. (2019)

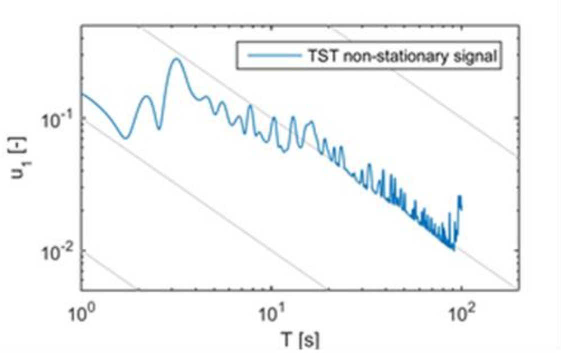

Figure 5: TST-B applied to non-stationary signal from Brouwer et al. (2019)

Both TST results show a large range where u_1 decays with the inverse of T. The signal is stationary in this range. For small sections, approximately $T < 14$ s in this case, the values of u_1 fall below the trend of the stationary range. In this region, the section is becoming shorter than the longest oscillation periods in the stochastic process. In those cases, the estimated standard deviation s of the signal segments underestimates the standard deviation of the stochastic process σ by a large factor due to a too short realisation length. For these short signal segments, the standard deviation s_m of mean is not a reliable measure of the uncertainty u_1 of mean.

A sudden rise in u_1 of the non-stationary signal can be observed for $T > 90$ s. This rise is called a 'hockey stick'. Since the uncertainty u_1 was calculated from the end of the signal, the hockey stick identifies a significant start-up transient for $t < t_{end} - 90$.

The optimal section that provides the most accurate mean value of the stationary process is identified just left of the onset of the hockey stick. In this case, around T equals 90 s. Since the uncertainty u_1 was taken from the end of the signal, the last 90 s of the signal represent the optimal section. The first 10 s of the signal should be excluded. The non-stationary realisation and its optimal section are shown in Figure 6.

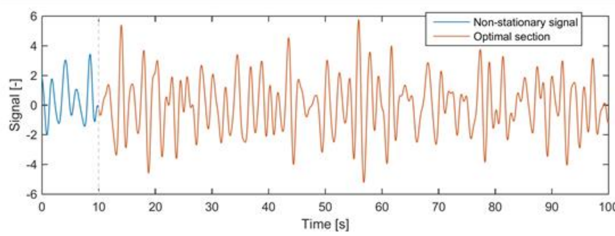

Figure 6: Realisation with added start-up disturbance and the optimal section indicated from Brouwer et al. (2019)

The mean value of the complete signal is 0.031, and its estimated standard uncertainty u_1 is 0.022. Therefore, the 95% confidence interval is constructed as (Brouwer et al. (2009)):

$$u_1=0.022;\ k_{95}=1.96;\ m=0.031\pm0.044$$

k_{95} is the coverage factor for a confidence level of 95%, and m is the sample mean.

The mean value of the optimal section is −0.0034, and its estimated standard uncertainty u_1 is 0.0105. Therefore, the 95% confidence interval is constructed as:

$$u_1 = 0.0105;\ k_{95} = 1.96;\ m = -0.003\pm0.021$$

The mean of the process μ, which is zero, lies well within both estimated confidence intervals. Removing the start-up transient has reduced the error in the sample mean by a factor of 10 and its corresponding uncertainty value by a factor of 2. The application of TST has increased the reliability of the mean value.

5. STEP BY STEP PROCEDURE

Although the mathematical processes behind the procedure can be challenging for 'non-experts', the process of obtaining the TST and the uncertainty U_m of mean is straightforward and is summarised in the following steps.

1. Obtain the time history ensuring that the sample rate is sufficient. For measurements in the laboratory or the field, the sample rate must be at least twice (often more) the highest frequency in the time history.

2. Ensure that the time history is as long as practicable.

3. TST-A

a) Select signal sections $[t_{\text{begin}}, t_{\text{begin}} + T]$ for series of signal length T. Calculate the standard uncertainty u_1 for each segment with:

$$u_1 = \sqrt{\frac{1}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) C_{xx,\text{biased}}(\tau) d\tau} \qquad (11)$$

The autocovariances $C_{xx,\text{biassed}}(\tau)$ are computed with the spectral method.

b) Plot graph with T on the x-axis and u_1 on the y-axis, both with logarithmic scales.

d) Identify any region where the slope differs from -1. This indicates non-stationary behaviour in the measurement, and this region should be removed from the mean estimation. Hockey sticks at large T indicate end effects.

4. TST-B

This procedure follows that of TST-A except the analysis begins at the end of the record with increasing values of T and are based on signal sections $[t_{\text{end}} – T, t_{\text{end}}]$. Hockey sticks at large T indicate start-up effects.

5. Random uncertainty of mean (RUM)

Removing end effects and start-up effects yields the stationary part of the signal. For this selected part computes the mean value. For the evaluation of the uncertainty, computes the autocovariance $C_{xx,\text{biassed}}(\tau)$with the spectral method and calculates the standard uncertainty u_1 of mean with Equation (11). Finally, the expanded random uncertainty U_m of mean is computed with Equation (12).

6. REFERENCES

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