ITTC Quality System Manual

Recommended Procedures and Guidelines

Guideline

Example for Uncertainty Analysis of Resistance Tests in Towing Tanks

7.5	Process Control
7.5-02	Testing and Extrapolation Methods
7.5-02-02	Resistance
7.5-02-02-02.1	Example for Uncertainty Analysis of Resistance Tests in Towing Tanks

Updated / Edited by	Approved
Quality Systems Group of 29th ITTC	29th ITTC 2021
Date 04/2020	Date 06/2020

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Example for Uncertainty Analysis of Resistance Tests in Towing Tank

1. PURPOSE OF PROCEDURE

The purpose of the procedure is to provide a real example in detail for performing uncertainty analysis in towing tank resistance tests that follow the ITTC Procedure 7.5-02-02-01, "Resistance Test" (2017a), in which David Taylor Model Basin (DTMB) model 5415 of a combatant with 5.72 m length is the example.

This procedure can be regarded as a supplement to the ITTC guideline 7.5-02-02-02, "General Guidelines for Uncertainty Analysis in Resistance Tests" (2014a) as well as provide quantitative results for extensive reference, since 41 institutions from 20 countries have participated in the Facility Bias World Wide Campaign, where two geosims of the DTMB model 5415 with 5.720 m and 3.048 m length, respectively, have been tested. The program was formulated at the 24th ITTC (2005). Results from 11 towing tanks were reported at the 27th ITTC (2014b) for the larger model, and the final results for the smaller model from 10 towing tanks were reported at the 28th ITTC (2017b)

The procedure has been revised for consistency with ITTC Procedure 7.5-02-02-02 (2014a) and Procedure 7.5-02-01-07 (2017c). In particular, the uncertainties are reported primarily as the expanded uncertainty U, rather than the standard uncertainty, u. A distinction is made in the difference between confidence and prediction limit. The unit of force is Newtons (N) rather than kilograms force (kgf) of the previous version. The reference list has been updated with the procedures from the 28th ITTC. Uncertainties related to extrapolation and full-scale prediction are not included in this procedure.

2. GENERAL DESCRIPTION OF THE EXAMPLE MODEL TEST

2.1 Test Model

A geosim hull model of surface ship, DTMB 5415, with 5.72 m length, made of Wawa wood, was manufactured with 5-axis CNC milling machine at China Surface Ship Research Centre (CSSRC) in late 2012.

The geometric parameters of the model given in Table 1 are calculated up to design draught through the numerical model for CNC manufacturing and regarded as theoretical values of this model. The tolerances of model hull lines were measured with a 3D Terrestrial Laser Scanner and satisfy the requirements by the ITTC Procedure 7.5-01-01-01, "Ship Models" (2017d), ±0.05 % L_{PP} or ±2.9 mm. A turbulence stimulation wire with diameter of 1.0 mm was mounted at the 19# station (5 % L_{PP} aft of the FP). The scale ratio of the model is λ = 24.824. Full-scale ship particulars are listed on the web page Simmon (2008).

2.1.1 Test Scheme

This model test was performed in the deep-water towing tank at CSSRC in early summer of 2013 and reported in Wu et al. (2013). The tank is 474 m long from the north to south end, 14 m wide in the test section and 7 m deep.

The measurands are the total resistance of model hull, running sinkage and trim at different Froude numbers. In each set of runs, Froude numbers increase successively from 0.10 to

0.45, which are set by towing speeds with the feedback control system of the towing carriage.

Table 1: Particulars of hull model

Symbol	Parameter	Model	Units
L_{PP}	Length between perpendiculars	5.7203	m
L_{WL}	Length on waterline	5.7258	m
B	Breadth on waterline	0.7666	m
T	Draught, even keel	0.2480	m
A_M	Midship section area	0.1557	m^2
A_W	Waterplane area	3.3968	m^2
S	Wetted surface area	4.8461	m^2
∇	Displacement volume	0.5517	m^3

In this example, a total of nine (9) repeat sets of runs were performed, for sake of simplicity, continuously and with the same instruments and installation by the same experienced engineers in the same way as the routine practice in the tank. Repeatability is as defined in JCGM (2008).

The dynamometer of type R63 measured resistance. The measurement at each speed is obtained by averaging the time history of the signal from the DAS (Data Acquisition System) in an interval of time, $\Delta t = n/f_s$,

$$R_T = (1/n)\sum_{i=1}^{n} R_i \tag{1}$$

where, f_s is the sampling rate, n the number of sampling data points, R_i the i-th data in the time history. In this example, f_s is selected as 50 Hz, Δt is at least 10 seconds and the low-pass cut-off frequency of filtering is 1.0 Hz. The standard deviation of a filtered time history is usually less than 0.2 % and then, the standard uncertainty of average of the sampling history will be less than 0.2 %/$\sqrt{500}$ = 0.009 %. That is, the uncertainty of one "reading" (the average value of a time history) from the DAS is negligible.

Two resistive-type linear-motion potentiometers are vertically mounted at the 1# station (2 mm aft of 1#) and 16# station (2 mm fore of 16#), respectively, and the strings are positioned 4294 mm apart for measuring the running trim and sinkage at the mid-station (10#).

The temperature of tank water is measured with three thermometers that are located at near end, middle area and far end of the tank, or, at 50 m, 200 m, and 300 m away from the north end, respectively. The mass density and viscosity of water are determined according to the ITTC Procedure 7.5-02-01-03, "Fresh Water and Seawater Properties" (2011).

The mid-sectional area of model hull is about 0.16 % of the tank sectional area. The blockage correction estimated by the Schuster formula from ITTC Procedure 7.5-02-02-01 (2017e) is negligible.

2.2 Data Reduction

2.2.1 Froude Number

The test was performed at three Froude numbers, Fr = 0.10, 0.28, and 0.41. The corresponding towing speeds were 0.749, 2.096, and 3.070 m/s, respectively. Towing speed deviates slightly from the nominal value as prescribed, e.g., V = 0.74888 m/s for Fr = 0.10 according to the Froude number calculation,

$$Fr = V/\sqrt{gL_{WL}} \tag{2}$$

where, L_{WL} is the waterline length and g the local acceleration of gravity in this example. The resistance measured at an actual towing speed, e.g., V = 0.748 m/s (Fr = 0.0999) for Fr = 0.1000, should be corrected to the nominal speed V = 0.74888 m/s. However, the difference is within the uncertainty estimate for the carriage speed of ±0.10 % (±0.00075 m/s) reported in a subsequent section 3.3.1.

2.2.2 Resistance

The total resistance coefficient formula is

$$C_{\mathrm{T}} = 2R_{\mathrm{T}}/(\rho S V^2) \tag{3}$$

For a correction to resistance from a difference in velocity of δV, C_{T} must be known as a function of Fr. For this model, the relationship is documented in Longo and Stern (2005). Over the range Fr = 0.10 ≤ 0.30, the value of C_{T} is relatively constant. From Equation (3), the correction for velocity over this range is then

$$\hat{R}_{\mathrm{T}} = R_{\mathrm{T}}(\delta V + V)^2/V^2$$

or for a small δV

$$\hat{R}_{\mathrm{T}} = R_{\mathrm{T}}(1 + 2\delta V)/V \tag{4}$$

At Fr = 0.45 from Longo and Stern (2005), C_{T} increases linearly, and the slope should be included in the estimate.

The frictional resistance coefficient by the ITTC-1957 model-ship correlation line from ITTC (2005, 2017a) is

$$C_{\mathrm{F}} = 0.075/(log_{10}Re - 2)^2 \tag{5}$$

where the Reynolds number is

$$Re = VL/\upsilon \tag{6}$$

L is selected as the waterline length in this example and υ the water kinematic viscosity. The effect of a little temperature variation on the model geometry is considered negligible.

For comparison between tests for the same test condition, the residuary resistance coefficient is applied

$$C_{\mathrm{R}} = C_{\mathrm{T}} - (1 + k)C_{\mathrm{F}} \tag{7}$$

where k is the form factor. The form factor is computed by the Prohaska method from ITTC Procedure 7.5-02-01-01 (2017a). By linear regression analysis from the following:

$$C_{\mathrm{T}}/C_{\mathrm{F}} - 1 = k + b(Fr^4/C_F) \tag{8}$$

where k is the intercept and b is the slope. From ITTC (2014b), k = 0.15 for Worldwide Campaign.

On the assumption of constant residuary coefficient for small changes, from Equation (7), the result is

$$\hat{C}_{\mathrm{T}} = C_{\mathrm{T}} + (\hat{C}_{\mathrm{F}} - C_{\mathrm{F}})(1 + k) \tag{9}$$

where $\hat{C}_{\mathrm{T}}$ and $\hat{C}_{\mathrm{F}}$ are at the revised test condition. For the Worldwide Campaign, the conditions for the results were re-computed at 15 °C from Equation (9).

2.2.3 Sinkage and Trim

The mean running sinkage is given by the following equation (ITTC 2005, 2017f):

$$z_{V\mathrm{M}} = (z_{V\mathrm{F}} + z_{V\mathrm{A}})/2 \tag{10}$$

where $z_{V\mathrm{F}}$ is running sinkage at the forward point (FP) and $z_{V\mathrm{A}}$ the running sinkage at the aft point (AP) from string potentiometers. From Longo and Stern (2005), the non-dimensional form for sinkage is

$$\sigma = 2z_{V\mathrm{M}}/(L_{\mathrm{PP}}Fr^2) \tag{11}$$

Running trim in pitch in radians is then defined from the ITTC (2005, 2017f) by

$$\theta_D = \tan^{-1}(z_{V\mathrm{F}} - z_{V\mathrm{A}})/L \quad (12)$$

For small pitch angles, Equation (12) is approximately

$$\theta_D \approx (z_{V\mathrm{F}} - z_{V\mathrm{A}})/L \quad (13)$$

where L is the distance between the strings of the string potentiometers. From Longo and Stern (2005), the non-dimensional trim is

$$\tau = 2(z_{V\mathrm{F}} - z_{V\mathrm{A}})/(L_{\mathrm{PP}} Fr^2) \quad (14)$$

However, for consistency with Equation (13), the length should be the distance between the strings of the string potentiometers.

2.3 Data of Resistance Measurement

The data of resistance measurements of nine (9) repeat tests, as examples, for Fr = 0.10, 0.28, and 0.41, are given in Table 2, corresponding to the nominal temperature 16.5 °C. The applied force during calibration from ITTC 7.5-01-03-01 (2017g) and OIML (2004) is

$$F = mg(1 - \rho_{\mathrm{A}}/\rho_{\mathrm{M}}) \quad (15)$$

where m is the nominal total mass of weights, g local gravity, ρ_{A} = 1.2 kg/m^3 air density, and ρ_{M} 8,000 kg/m^3 the conventional density of the weights. At CSSRC, local g is 9.7946 m/s^2. The value of local g as computed from ITTC (2017g) is 9.79439 ±0.00020 m/s^2. An estimate of local g anywhere in the world was provided by Physikalisch-Technische Bundesanstalt (PTB), Braunschweig , Germany, at their web page: http://www.ptb.de/cartoweb3/SISproject.php. For the calibration stand, the mass is the sum of the weights:

$$m = \sum_{i=1}^{n} m_i \quad (16)$$

From Equation (4), the resistance correction to the nominal Fr = 0.10 for the velocity differential of 0.001 m/s at the average load of 5.343 N is 1.34 or an increase of 34 %.

Table 2: Data of resistance measurement

Total Resistance (16.5 °C)_April_30_2013			
R_{T} (N)	Fr = 0.10	Fr = 0.28	Fr = 0.41
Run #1	5.298	44.64	148.06
Run #2	5.288	44.21	148.03
Run #3	5.425	44.64	147.62
Run #4	5.386	44.64	148.22
Run #5	5.416	44.68	146.79
Run #6	5.327	44.64	146.96
Run #7	5.347	44.90	146.98
Run #8	5.327	44.46	146.80
Run #9	5.269	44.82	147.51
Average	5.343	44.62	147.44
Std. Dev.	0.056	0.20	0.58
g = 9.7946 m/s^2			

3. UNCERTAINTY EVALUATION

3.1 Model Ballasting

The model hull with the instruments mounted on-board is ballasted to its displacement mass, Δ, that is determined by its nominal displacement volume, ∇, and the mass density of towing tank water, ρ, at the temperature (16.5 °C) measured the day before test,

$$\Delta = \rho\nabla = 551.073 \text{ kg} \quad (17)$$

As a result, the model was measured 551.0 kg by a digital scale with an expanded uncertainty of ±0.5 kg or ±0.091 % relative uncertainty.

The uncertainty in displacement may also be estimated from the uncertainty in the location of the waterline. The expanded uncertainty is

$$U_\Delta = \rho A_W U_T \quad (18)$$

where ρ is the density of the water and U_T is the uncertainty in the draught or location of the waterline. As an estimate with U_T = 1.0 mm, water density of 998.86 kg/m^3, and A_W = 3.3968 m^2 from Table 1, the estimated uncertainty in displacement is ±3.4 kg or ±0.62 %.

The uncertainty in wetted surface area may also be estimated from the displacement. From ITTC (2014c), the uncertainty is

$$U_S/S = (2/3)U_\Delta/\Delta \quad (19)$$

From the above uncertainty in displacement, the expanded relative uncertainty in S is ±0.41 %.

Temperature variation of the tank water with time and location is within ±0.10 °C during the model tests, with the uncertainty in the thermometer calibration of ±0.20 °C, combined and expanded uncertainty in water temperature during the test is ±0.22 °C. From ITTC 7.5-02-01-03 (2011), the water density is 998.863 ±0.037 kg/m^3 (±0.0037 %).

The static trim and heel angles of the hull are trimmed to be within ±0.05° and ±0.15°, respectively. These uncertainties are assumed negligible to the wetted surface area and resistance of the hull model.

3.2 Model Installation

Uncertainties from installation related to the hull resistance are mainly attributed to the alignment between the longitudinal centrelines of hull, resistance dynamometer, towing guide and towing tank/towing carriage rails.

In this example, the misalignment between the centreline of hull and the towing force of dynamometer is estimated to be within ±0.10°, which results in a negligible uncertainty in the model hull resistance measurement. In general, to evaluate the uncertainty of the model hull resistance is not practical due to the misalignment of the hull and tank except if a suitable side-force measurement dynamometer is installed.

3.3 Instrument Calibration

3.3.1 Tachometer for Towing Speed

The tachometer for towing carriage speed is mainly composed of a trailing wheel and encoder and the towing carriage is calibrated regularly. The carriage speed is

$$V = \pi D N \quad (20)$$

where D is the diameter of the wheel, N is the rotational rate from the encoder and timing system. The uncertainty of towing speed for the range 0.75~3.5 m/s can be estimated as 0.10 %, although the uncertainty is less than 0.10 % for speeds greater than 1.0 m/s.

From Equation (2) and the law of propagation of uncertainty, the relative uncertainty in Froude number is

$$U_{Fr}/Fr = \sqrt{(U_V/V)^2 + (U_L/(2L))^2} \quad (21)$$

For U_V = ±0.10 % and U_L = ±0.05 %, the relative expanded uncertainty in Fr is ±0.10 %.

From Equation (6), the expanded uncertainty in Reynolds number is as follows:

$$U_{Re}/Re = \sqrt{(U_V/V)^2 + (U_L/(L))^2 + (U_\upsilon/(\upsilon))^2} \quad (22)$$

where kinematic viscosity is determined by the water temperature from

$$U_v/v = (\partial v/\partial t)U_t/v \quad (22a)$$

From the previous values of V and L and the uncertainty in viscosity of U_υ = ±0.57 % from ITTC (2011) and an uncertainty in temperature of 0.22 °C, the expanded uncertainty in Reynolds number is ±0.58 %. The uncertainty is dominated by viscosity.

3.3.2 Dynamometer for Resistance

The dynamometer was calibrated before model tests according to the ITTC Procedure 7.5-01-03-01 (2017g). The calibration range is chosen as not less than 1.5 times the maximum of hull drag that is estimated beforehand. In this example, the maximum load is selected as 32 kg or 313 N. Eleven loads are implemented by weights and randomly applied three times for each load as shown in Figure 1.

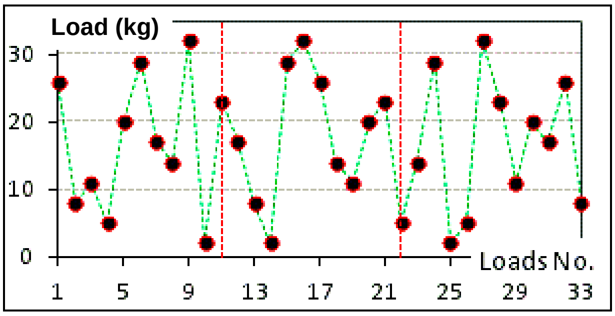

Figure 1: Calibration loadings for dynamometer

The fitting curve for predicting force is obtained by linear regression,

$$F(\mathrm{N}) = 81.800 \times Voltage(\mathrm{V}) \quad (23)$$

with a standard error of estimate, *SEE*, of

$$SEE = 0.0852\ \mathrm{N} \quad (24)$$

which will result in a standard uncertainty of resistance measurement,

$$u_{R_\mathrm{T}} = 0.0852\ \mathrm{N} \quad (25)$$

Application of *SEE* as an uncertainty estimate under-estimates the uncertainty. A better method is the prediction limit from linear regression analysis as described by ITTC (2017g).

The average resistances measured, in this example, at Fr = 0.10, 0.28, and 0.41 are 5.343, 44.62, and 147.44 N, respectively. The relative expanded uncertainties with k = 2 corresponding to the calibration component are about 3.2 %, 0.38 %, and 0.12 %, respectively. At the velocity correction for Fr = 0.10, load correction of 34 % is significantly larger than the uncertainty in measurement from the dynamometer.

Additionally, the weights for loading are rated the OIML Class M_2 (2004), which have a tolerance of 0.015 %. With the tolerance applied as the expanded uncertainty, such uncertainty component related to weights is negligible in the total resistance measurement, R_T.

The dynamometer is checked after tests by successively loading and unloading weights of 5 kg, 10 kg, 15 kg, 20 kg, 25 kg, and 30 kg. The deviation of checking result from Equation (23) is 0.014 % and negligible, which confirms the reproducibility of dynamometer measurements during the tests. Reproducibility is as defined in JCGM (2008).

For the relative uncertainty in C_T from Equation (3) during testing and the law of propagation on uncertainty, the relative uncertainty is

$$U_{C_\mathrm{T}}\Big/C_T = \sqrt{\begin{matrix}(U_{R_\mathrm{T}}/R_\mathrm{T})^2 + (U_\rho/\rho)^2 \\ +(2\,U_V/V)^2 + (U_S/S)^2\end{matrix}} \quad (26)$$

where density is determined by the water temperature from

$$U_\rho/\rho = (\partial\rho/\partial t)U_t/\rho \quad (26a)$$

The density and its derivative as a function of temperature are listed in ITTC Procedure 7.5-02-01-03 (2011).

3.3.3 Devices for Sinkage and Trim

On basis of the potentiometer specification, the expanded uncertainty is 0.10 %×400 mm = 0.40 mm. Since the same specification is applied, the uncertainty in measurement is correlated. From Equation (10), the expanded uncertainty is

$$U_{zVM} = (U_{zVF} + U_{zVA})/2 = U_z \quad (27)$$

Thus, the measured uncertainty in sinkage is ±0.40 mm. For a better estimate of the uncertainty, the string pots should be calibrated. Equation (27) will also apply to calibration string potentiometers since they would be calibrated against the same calibration reference, and the uncertainty result will be correlated.

Similarly, from Equation (13) excluding the uncertainty in L, the uncertainty in trim will be zero (0) due to the difference.

$$U_{\theta D} = (U_{zVF} - U_{zVA})/L = 0 \quad (28)$$

where L = 4294 mm. The relative uncertainty in trim is then primarily from the uncertainty in distance between the string potentiometers.

$$U_{\theta D}/\theta_D = U_L/L \quad (29)$$

The estimated deviation is ±2.0 mm for the distance (4294 mm) between potentiometers. From Equation (29), the relative uncertainty is ±0.046 %. The verticality within ±0.1° of their installation will have a negligible contribution to the uncertainties of sinkage and trim measurements.

No analytic relationship exists between the ship model resistance and running trim and sinkage. They are among the parameters for validating CFD, running trim and sinkage can also provide indispensable information for analysing repeat tests and performing inter-laboratory comparison of resistance tests.

3.3.4 Thermometer for Water Temperature

The digital thermometer has a display resolution of 0.1 °C. From its technical specification, the uncertainty of the thermometer is quoted as 0.2 °C. From the daily variation in water temperature and the calibration uncertainty, the estimated combined and expanded uncertainty is ±0.22 °C as reported in a previous section on water density.

In this example, the water temperatures measured during nine repeat sets of tests are shown in Figure 2. The mean temperature is 16.5 °C.

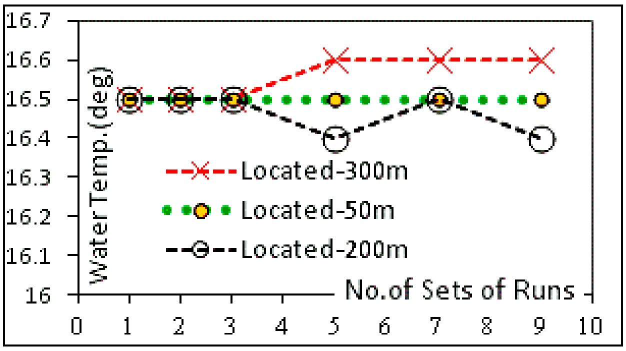

Figure 2: Water temperature measured in tank

For water at 16.5 ±0.22 °C, the water kinematic viscosity from ITTC Procedure 7.5-02-01-03 (2011) is (1.0950 ±0.0063) ×10^{-6} m^2/s or ±0.56 %.

The relative uncertainty in the friction coefficient for each tow speed is estimated by

$$U_{CF}/(C_F) = 0.87(U_\upsilon/\upsilon)/(\log_{10} Re - 2) \quad (30)$$

In this example, at *Fr* = 0.10, 0.28, and 0.41, the values of *Re* are 3.9×10^6, 1.1×10^7, and 1.6×10^7, respectively. Then, the relative uncertainties of C_F are 0.11, 0.098, and 0.094 % for *Fr* = 0.10, 0.28, and 0.41, respectively.

3.4 Repeat Tests

3.4.1 Resistance

The means, standard deviations (*s*), minimums and maximums of measured resistance at water temperature 16.5 °C in 9 repeat tests are given Table 4 for all three *Fr*. No outliers are observed.

The mean of repeat measurements is usually adopted as the best estimate for a measurand. The standard uncertainty component of the mean from *n* repeat tests with a standard deviation, *s*, is estimated from JCGM (2008) and ITTC Procedure 7.5-02-01-01 (2014a) by

$$u_A = s/\sqrt{n} \tag{31}$$

The expanded uncertainty at the 95 % confidence level by the Type A method is then for a specific test

$$U_A = ku_A \tag{32}$$

where usually *k* = 2 but for the Student-*t* *k* = 2.3 for 9 samples. However, if the result is applied to a future event such a applying the model test result to a full-scale ship or calibration data to as test, then the prediction limit at the 95 % level applies as follows from ITTC Procedure 7.5-02-01-07 and 7.5-01-03-01 (2017c, g) and Devore (2008):

$$U = ks\sqrt{1 + 1/n} \tag{33}$$

For 9 repeat tests with the Student-*t* as the coverage factor, Equation (33) becomes:

$$U = 2.43\ s \tag{34}$$

From Table 4, the uncertainty of measurement in resistance tests for this hull model is estimated at nominally ±2.5 % for *Fr* = 0.10, ±1.0 % for *Fr* = 0.28, and *Fr* = 0.41, respectively, at 95 % prediction limit.

Table 3: Statistical analysis of repeat measurement for resistance

Fr	R_T (N)_(16.5 °C)			
	Mean	*s* (%)	Min	Max
0.10	5.337	1.04	5.269	5.464
0.28	44.62	0.45	44.46	44.90
0.41	147.44	0.39	146.79	148.22

Table 4: Uncertainty of repeat measurements for resistance

Fr	R_T (N)_(16.5 °C)			
	Mean	u_A (%)	*s* (%)	*U* (%)
0.10	5.337	0.35	1.04	2.53
0.28	44.62	0.15	0.45	1.09
0.41	147.44	0.13	0.39	0.95

3.4.2 Running Sinkage

No outlier was observed among all the running sinkages measured in nine repeat sets of runs. The uncertainty analysis for direct measurement of sinkage is given in Table 5. The uncertainty of measurement is estimated at nominally ±1.0 mm. No further detail is provided for the running sinkage measurement in this procedure.

Table 5: Uncertainty of repeat measurement for running sinkage

Fr	Sinkage (mm)_(16.5 °C)			
	Mean	u_A	*s*	*U*
0.10	-1.08	0.11	0.33	0.80
0.28	-9.83	0.13	0.40	0.97
0.41	-24.86	0.10	0.31	0.75

3.4.3 Running Trim

No outlier is observed among all the running trims measured in nine repeat sets of runs. The uncertainty analysis for direct measurement of trim is given in Table 6. All the running trims are less than one degree and the uncertainty of repeat measurement is much less than 0.05°. Considering the accuracy of static trim is estimated within ±0.05°, the repeat uncertainty of running trim is not significant. No further detail is provided for the running trim measurement in this procedure.

Table 6: Uncertainty of repeat measurement for running trim

Fr	Trim (degrees)_(16.5 °C)			
	Mean	u_A	*s*	*U*
0.10	-0.004	0.005	0.015	0.036
0.28	-0.099	0.003	0.008	0.019
0.41	0.392	0.004	0.013	0.032

3.5 Combination of Uncertainty Components of Resistance Measurement

Based on the above analysis, all the significant components of uncertainty in resistance measurement are summarized and combined through RSS (Root-Sum-Square) as listed in the following Table 7 through Table 9.

Table 7: Combination of uncertainty in measurement for resistance (Fr = 0.10)

a. Resistance, R_T

R_T (*Fr* = 0.10, 16.5 °C)	Type	*U* (%)	Remark
Dynamometer	B (ν=32)	3.2	dominant
Repeat test, Deviation	A (*n*=9)	2.5	secondary
Combined for single test (prediction limit)		4.1	U_c
Repeat test, Deviation of mean	A (*n*=9)	0.80	U_A
Combined for test avg (confidence limit)		3.3	U_c

b. Total resistance coefficient, C_T

C_T (*Fr*=0.10, 16.5 °C)	Type	*U* (%)	Remark
Wetted area	B	0.410	negligible
Speed	B	0.200	negligible
Water den-	B	0.004	negligible
Dynamometer	B (ν=32)	3.189	dominant
Repeat test, Deviation	A (n=9)	2.540	secondary
Combined for single test (prediction limit)		4.103	U_c
Repeat test, Deviation of mean	A (*n*=9)	0.804	U_A
Combined for test avg (confidence limit)		3.321	U_c

Table 8: Combination of uncertainty in measurement for resistance (Fr = 0.28)

a. Resistance, R_T

R_T (Fr = 0.28, 16.5 °C)	Type	U (%)	Remark
Dynamometer	B (v=32)	0.38	secondary
Repeat test, Deviation	A (n=9)	1.1	dominant
Combined for single test (prediction limit)		1.2	U_c
Repeat test, Deviation of mean	A (n=9)	0.34	U_A
Combined for test avg (confidence limit)		0.51	U_c

b. Total resistance coefficient, C_T

C_T (Fr=0.28, 16.5 °C)	Type	U (%)	Remark
Wetted area	B	0.410	secondary
Speed	B	0.200	negligible
Water density	B	0.004	negligible
Dynamometer	B (v=32)	0.382	secondary
Repeat test, Deviation	A (n=9)	1.091	dominant
Combined for single test (prediction limit)		1.243	U_c
Repeat test, Deviation of mean	A (n=9)	0.345	U_A
Combined for test avg (confidence limit)		0.688	U_c

Table 9: Combination of uncertainty in measurement for resistance (Fr=0.41)

a. Resistance, R_T

R_T (Fr = 0.41, 16.5 °C)	Type	U (%)	Remark
Dynamometer	B (v=32)	0.12	negligible
Repeat test, Deviation	A (n=9)	0.95	dominant
Combined for single test (prediction limit)		0.96	U_c
Repeat test, Deviation of mean	A (n=9)	0.30	U_A
Combined for test avg (confidence limit)		0.32	U_c

b. Total resistance coefficient, C_T

C_T (Fr=0.41, 16.5 °C)	Type	U (%)	Remark
Wetted area	B	0.410	secondary
Speed	B	0.200	negligible
Water density	B	0.004	negligible
Dynamometer	B (v=32)	0.116	negligible
Repeat test, Deviation	A (n=9)	0.952	dominant
Combined for single test (prediction limit)		1.062	U_c
Repeat test, Deviation of mean	A (n=9)	0.301	U_A
Combined for test avg (confidence limit)		0.559	U_c

3.6 Combination of Uncertainty Components of Measurement of Running Sinkage

Based on the above analysis, all the significant components of uncertainty in measurement of running sinkage are summarized and combined through RSS as listed in the following Table 10 through Table 12.

Table 10: Combination of uncertainty in measurement for running sinkage (*Fr* = 0.10)

Sinkage (Fr=0.10)	Type	U (mm)	Remark
Potentiometer	B	0.40	secondary
Repeat test, Deviation	A (n=9)	0.80	dominant
Combined for single test (prediction limit)		0.89	Uc
Repeat test, Deviation of mean	A (n=9)	0.22	UA
Combined for test avg (confidence limit)		0.46	Uc

Table 11: Combination of uncertainty in measurement for running sinkage (*Fr* = 0.28)

Sinkage (*Fr*=0.28)	Type	*U* (mm)	Remark
Potentiometer	B	0.40	secondary
Repeat test, Deviation	A (*n*=9)	0.97	dominant
Combined for single test (prediction limit)		1.0	U_c
Repeat test, Deviation of mean	A (*n*=9)	0.26	U_A
Combined for test avg (confidence limit)		0.46	U_c

Table 12: Combination of uncertainty in measurement for running sinkage (*Fr*=0.41)

Sinkage (*Fr*=0.41)	Type	*U* (mm)	Remark
Potentiometer	B	0.40	secondary
Repeat test, Deviation	A (*n*=9)	0.75	dominant
Combined for single test (prediction limit)		0.85	U_c
Repeat test, Deviation of mean	A (*n*=9)	0.20	U_A
Combined for test avg (confidence limit)		0.48	U_c

4. REPORT OF UNCERTAINTY OF RESISTANCE MEASUREMENT

The total resistances of the model ship measured in water temperature of 16.5 °C can be expressed as the following.

- Measurement for a single test at the prediction limit

$$\hat{R}_T = R_T[1 \pm ks\sqrt{1 + 1/n}] \tag{35}$$

- Measurement for average of repeat tests at the confidence limit

$$\hat{R}_T = R_T[1 \pm ks/\sqrt{n}] \tag{36}$$

where the coverage $k = 2$ corresponds to the confidence level of 95% and n the number of repeat tests. The coverage factor may also be the Student-*t*. Usually, only two significant figures are retained in the expression of uncertainty values.

The measurement results (mean values of repeat tests) of this example are given in Table 13 and the corresponding non-dimensional values, i.e., the total resistance coefficients given in Table 14.

Table 13: Resistance with expanded uncertainty (*k* = 2) measured in fresh water of 16.5 °C

Total Resistance at 16.5 °C			
Fr	Resistance (N)	Sinkage (mm)	Trim (de-
0.10	5.34 ±0.18 (±3.3 %)	-1.08 ±0.46	-0.004 ±0.051
0.28	44.62 ±0.31 (±0.69 %)	-9.83 ±0.46	-0.099 ±0.050
0.41	147.44 ±0.84 (±0.56 %)	-24.86 ±0.48	0.390 ±0.051

Table 14: Resistance coefficient with expanded uncertainty (*k* = 2) in fresh water of 16.5 °C

Total Resistance Coefficient at 16.5 °C			
Fr	C_T (10^{-3})	Sinkage (mm)	Trim (degrees)
0.10	3.94 ±0.13 (±3.3 %)	-1.08 ±0.46	-0.004 ±0.051
0.28	4.193 ±0.029 (±0.69 %)	-9.83 ±0.46	-0.099 ±0.050
0.41	6.462 ±0.036 (±0.56 %)	-24.86 ±0.48	0.390 ±0.051

From Equation (9), the resistance coefficient at temperature of 16.5 °C in Table 14 is converted to the nominal temperature 15 °C of fresh water in Table 15.

The running sinkage and trim will provide important information for intra- and inter-laboratory comparison although, as shown in this example, their uncertainties from repeat measurements by the Type A method are not significant. Most of the uncertainty is from the Type B method.

For sinkage, most of the uncertainty is from the string potentiometers at all speeds in mm. The uncertainty in the average ranges from ±0.46 to ±0.48 mm with the string pot specification of ±0.40 mm. Likewise, the uncertainty in trim is from the uncertainty in the measurement of ±0.050° rather than the repeatability.

Additionally, with reference to this example, planning of routine resistance tests can focus on the dominant sources of uncertainties for an improvement in the quality of the test.

Table 15: Resistance Coefficient with expanded uncertainty (*k* = 2) in fresh water of 15 °C

Total Resistance Coefficient at 15 °C			
Fr	C_T (10^{-3})	Sinkage (mm)	Trim (degrees)
0.10	3.97 ±0.13 (±3.3 %)	-1.08 ±0.46	-0.004 ±0.051
0.28	4.216 ±0.029 (±0.69 %)	-9.83 ±0.46	-0.099 ±0.050
0.41	6.483 ±0.036 (±0.56 %)	-24.86 ±0.48	0.390 ±0.051

In this example, the uncertainty of resistance measurement depends highly on the uncertainty of the dynamometer calibration and the number of repeat tests. Specifically, for this test at *Fr* = 0.10, the dynamometer calibration is the dominant contributor to the uncertainty while at *Fr* = 0.41, the standard deviation of the repeat tests dominates in the prediction limit. However, in the confidence interval for the average, the wetted surface area uncertainty at ±0.41 % is dominant at *Fr* = 0.41 in comparison to the combined uncertainty of ±0.56 % for the total resistance coefficient, C_T.

The uncertainty in the wetted surface area, *S*, for the total resistance coefficient, C_T, was computed from the uncertainty in uncertainty in the waterline location from Equations (18) and (19) rather than the model weight uncertainty from a single measurement. A single measurement before the test may not be representative of the weight during testing. Another method is to compute the wetted surface area from laser measurements of the model hull.

Uncertainty analysis assists in the identification of the proper instrumentation and adequate number of repeat tests to meet with a desired accuracy of measurement. Any improvement in an uncertainty estimate should focus on the dominant source of uncertainty.

5. LIST OF SYMBOLS

5.1 English

A_M	Midship section area	m^2
A_W	Water-plane area	m^2
B	Breadth at waterline	m
C_F	Frictional resistance coefficient, Equation (5)	1
C_R	Residuary resistance coefficient, Equation (7)	1
C_T	Total resistance coefficient, Equation (3)	1
D	Diameter	m
F	Force, Equation (15)	N
Fr	Froude number, Equation (2)	1
g	Local acceleration of gravity	m/s^2
k	Coverage factor, usually $k = 2$	1
k	Form factor	1
L	Length	m
L_{PP}	Length between perpendiculars	m
L_{WL}	Length on waterline	m
m	Mass	kg
N	Rotational rate	rad/s
n	Number of samples	1
R_T	Total resistance	N
Re	Reynolds number, Equation (6)	1
S	Wetted surface area	m^2
s	Standard deviation	
T	Draught	m
t	Temperature	°C
U	Expanded uncertainty, $U = ku$	
u	Standard uncertainty	
V	Velocity	m/s
z_{VA}	Aft running sinkage	m
z_{VF}	Forward running sinkage	m
z_{VM}	Mean running sinkage, Equation (10)	m

5.2 Greek

Δ	Displacement	kg
θ_D	Running trim, Equations (12) (13)	rad
μ	Absolute viscosity	Pa·s
υ	Kinematic viscosity, $\upsilon = \mu/\rho$	m^2/s
ρ	Density	kg/m^3
σ	Non-dimensional sinkage, Equation (11)	1
τ	Non-dimensional trim, Equation (14)	1

5.3 Other

∇	Volumetric displacement, $\nabla = \Delta/\rho$	m^3

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